Evaluation of the Productive Process by Means of Control Charts in the Presence of Volatility

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ABSTRACT

The main purpose of this research work is to verify the stability of a productive process in the presence of the effects of autocorrelation and volatility so that these characteristics may be captured by a joint forecast model which produces residuals to be used in a control chart. Also, the effects of these factors will be analyzed to verify the impact in the performance of the control chart. The autoregressive integrated moving average (ARIMA) model was used along with the autoregressive conditional heteroskedasticity model (ARCH) to deal with the autoregression and volatility present in the data. The process stability was analyzed by means of control charts applied to the residuals coming from the joint model. The AR (1) - ARCH (1) model shows that the use of an appropriate forecasting model brings significant contributions to the control chart performance.

KEY WORDS: Statistical control process, Autocorrelated data, Volatility in industrial processes, ARIMA models, ARCH models.

1 INTRODUCTION

The assumptions required to implement a control chart are that the sample data to be analyzed comes from a population of independent data, follows the Normal distribution and is independent and identically distributed (*iid*) [1], [2], [3], [4], [5].

Traditionally, linear models such as the autoregressive integrated moving average (ARIMA) are used to remove serial correlation. The residuals originated from these models are used to evaluate the stability of the productive process in study.

For the appropriate application of control charts these residuals must possess some properties, such as presenting normality and independency, as well as having zero mean and constant variance, that is, being *white noise*.

However, as noted by several authors [6], [7], [8], [9], the residuals of a linear model may be conditionally heteroskedastic. In this case, the conditional variance of the residuals can be investigated by an autoregressive conditional heteroskedasticity model (ARCH). The ARCH models proposed by Engle [10] and Bollerslev [11], explain volatility using the past squared residual values originated from a linear prediction model. They show that if these characteristics are neglected, there are consequences in terms of the quality of the parameter estimates and consequently in the forecasting values and residuals.

Therefore, the main purpose of this study is to verify the stability of a productive process in the presence of autocorrelation and time-varying volatility. It is necessary to capture these characteristics by means of a joint ARIMA - ARCH model, so that the residuals that come from this forecasting model may be used by a control chart to evaluate the stability process. Besides, the effects of these factors on the performance of control charts when a productive process presents volatility were also studied.

2 METHODOLOGY

The methodological steps to be followed in order to obtain a residual series free of autocorrelation and heteroskedasticity to apply the control charts are:

STEP 1: Descriptive statistics in order to understand the behavior of the variable under study;

STEP 2: Linear modeling - *ARIMA (p, d, q)*, using the B-J methodology [12] in order to remove serial correlation and analyze the series residuals;

STEP 3: Heteroskedasticity residuals analysis in order to verify the presence of autoregressive conditional heteroskedasticity using the ARCH-LM test proposed by Engle [10];

STEP 4: Non-linear modeling - *ARCH (p)*, the joint modeling is done by ARIMA-ARCH models considering the level and volatility effects of the series [13], which will be simultaneously estimated by means of the Eviews 6.0 software;

STEP 5: Application of mean control charts for individual measures in the residuals free of autocorrelation and heteroskedasticity effects, using the Statistica 7.0 software.

The evaluation of stability will be done using not only the sample out of control limits [14], but also the run test rules will be applied [15], [16];

STEP 6 – The ARCH effect evaluation in order to verify the performance of control charts and the Granger causality direction according to [17]. The test will be applied to the original series, and to the residuals derived from the ARIMA model and the model set ARIMA-ARCH.

3 RESULTS ANS DISCUSSION

This research focuses on the analysis of stability of the variable acidity index of soybean oil because the company warns that there may be economic losses if the quality standards are not met, as well as because their lipid properties as [18], point out.

Data were collected from April 1st, 2004 to June 30th, 2009, totalizing 960 daily observations.

Descriptive statistics

Figure 1 shows that the acidity index is stable around the mean, with some extreme points and apparently with great variability. Furthermore, we observe the presence of outliers, being that the most pronounced is in the 796th observation on August 14th, 2008.

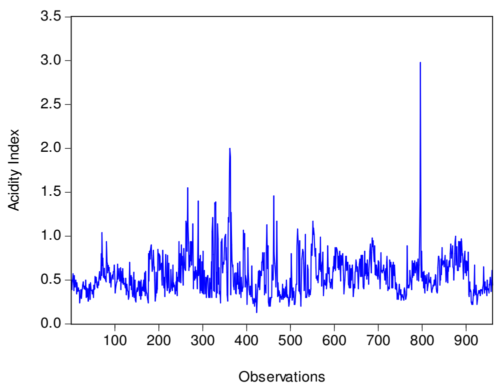

FIGURE 1 - Original series of the acidity index of the soybean oil variable

The descriptive analysis shows that the mean is not very representative because the variation coefficient (42.841) is very close to 50%. The skewness coefficient (2.329) is significantly different from zero and kurtosis (17.046) is significantly greater than three, which is indicative of time-varying volatility and volatility clusters [19]. In order to test the null hypothesis that the sample follows a normal distribution, the Jarque-Bera test was performed. The JB statistics is 8757.675 (p-value < 0.01).

Stationarity was assessed using Augmented Dickey Fuller (1981), Elliott-Rothenberg-Stock (1996) and Kwiatkowski-Phillips-Schmidt-Shin (1992) tests, all of them permitting to conclude that the series in levels is stationary.

Modeling step of acidity index of soybean oil

Among the competing models found to evaluate the acidity index of soybean oil and based on the residuals analysis taking into account the AIC and BIC criteria (and the parsimony criterion), the most appropriate model is the first order autoregression – AR(1). To test statistically for the presence of conditional heteroskedasticity, the ARCH-LM test proposed by Engle [10], was performed on the residuals obtained out of the AR (1) model (Table 1).

TABLE 1 - ARCH-LM test applied to the AR (1) residuals to assess conditional heteroskedasticity

ARCH test	Statistics	Degrees of freedom	p-value
F-statistic	174.6912	$F(1,956)$	0.0000
TR^2	148.0105	χ^2 (1)	0.0000

Table 1 shows that the null hypothesis of no ARCH effects is rejected in both tests. Thus, the squared residuals of the AR (1) model are considered to be conditionally heteroskedastic.

Table 2 presents the joint model considering the effect of the first two moments simultaneously.

TABLE 2 – Estimation of coefficients, standard-error, Z statistic and p-value of AR-ARCH model to the acidity index of soybean oil

Method: ML – ARCH (Marquardt) – Normal distribution				
Mean conditional equation				
	Coefficient	Standard error	Z statistics	p-value
Constant	0.558189	0.012347	45.20901	0.000
AR (1)	0.570753	0.022296	25.59881	0.000
Conditional variance equation				
Constant	0.023587	0.001274	18.52063	0.000
ARCH (1)	0.497435	0.044483	11.18261	0.000

The model that describes the mean and the volatility is described by a joint AR (1) - ARCH (1) model presenting parameters statistically significant and the residual series behaving as *white noise*. The model validation was performed by examining statistics such as skewness, kurtosis as well as normality and residual independence. Other competing models such as *e.g.* GARCH, EGARCH and TARCH were tested and did not produce better results than the ARCH (1) model.

In order to test for conditional heteroskedasticity in the residuals of the final model, the ARCH-LM test was applied to the residuals of the AR (1)-ARCH (1) model (Table 3).

TABLE 3 - ARCH-LM test applied to the AR (1) - ARCH (1) residuals to assess conditional heteroskedasticity

ARCH test	Statistics	Degrees of freedom	p-value
F-statistic	0.354743	$F(1,956)$	0.5516
TR^2	0.355353	χ^2 (1)	0.5511

Looking at the statistics presented in Table 3, we do not reject the null hypothesis of no ARCH effects on the final residuals obtained from the joint model. So, the final residuals now meet the requirements to be analyzed by control charts.

Process stability analysis

The process stability analysis at this moment is represented by the X-bar chart for individual measurements. To make a comparison, first the original variable will be analyzed and then we analyze the residual series that comes out of the joint AR (1) - ARCH (1) model, so that we can notice the changing behavior of the control charts.

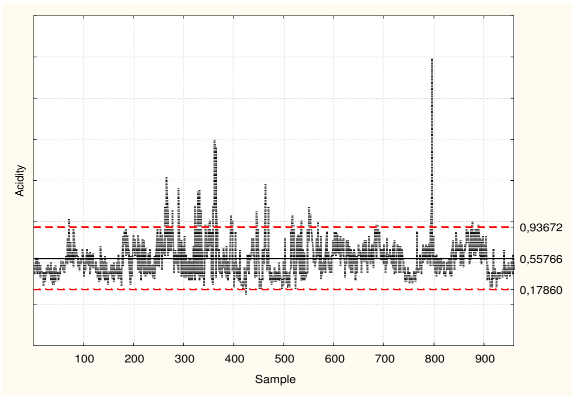

FIGURE 2 – X-bar control chart for individual measurements of acidity index of soybean oil

In Figure 2 there are many points out of the control limits that do not follow a common pattern in control charts causing great instability in the productive process.

Figure 3 presents the monitoring of the residuals from the joint AR (1) – ARCH (1) model. This control chart shows much less points out of the control limits and the process is noticeably more stable.

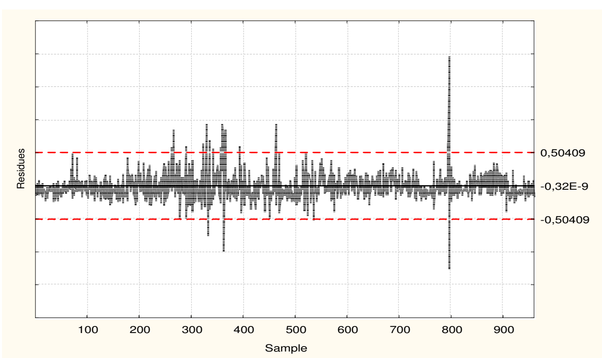

FIGURE 3 – X-bar control chart for individual measurements of residuals from the joint AR (1)-ARCH (1) model of the acidity index variable of soybean oil

Figure 3 shows that there are minor indications of instability and that there are changes in the control chart limits, showing the absence of the effect of autocorrelation and conditional heteroskedasticity.

ARCH effect evaluation

The Granger causality test is applied to verify if a variable improves its forecasting estimation. Thus, the test is used to check the direction of causality. The variables involved in this test are the series in levels, the residuals of the autoregressive model AR (1) and the residuals of the joint model AR (1) - ARCH (1).

TABLE 4 - Granger causality tests applied to the series in levels, AR (1) residuals and the joint model AR (1)-ARCH (1) residuals

Null hypothesis	*F*-Statistic	Probability
a_t does not Granger cause AC	15.5235	8.7E-05
AC does not Granger cause a_t	10.5407	0.00121
e_t does not Granger cause AC	15.5235	8.7E-05
AC does not Granger cause e_t	10.5650	0.00119
e_t does not Granger cause a_t	10.5407	0.00121
a_t does not Granger cause e_t	10.5650	0.00119

*AC – Acidity index of soybean oil in level

** a_t – Residuals from the AR (1) model

*** e_t – Residuals from the joint AR (1) – ARCH (1) model

These tests use 958 observations and allow us to corroborate the findings of the control charts. The null hypothesis is rejected in all cases and both the residuals of the AR and of the AR-ARCH model have an influence on each other. So, it is appropriate to use the model that can explain the data series in levels and in its variability to obtain a better performance of control charts.

4 CONCLUSIONS

The joint AR-ARCH model shows that an appropriate forecasting model brings a great contribution to the performance of control charts to monitor the stability of industrial variables.

It is important to highlight that if the stability analysis of the production process had been applied directly on the original variable, five hundred eighty-five (585) sample points would be pointed out as a possible source of instability. On the other hand, using the joint AR (1) - ARCH (1) model, the number of possible source of instability points was reduced to two hundred sixty-two (262). So, there is a substantial reduction of effort to the control team because each detected point which signalizes instability in the system should be identified and tracked so that future actions may be taken.

This research is important to the industry in order to present an alternative approach to traditional techniques of statistical control process. The volatility that, until now, has been treated as a problem in modeling may bring a new adequate way to understand industrial processes and proper use of control charts in autoregressive situations.

The test of Granger causality shows that there is a bi-directional effect between the level of acidity, the residuals of AR (1) and the residuals of AR (1) - ARCH (1). This direction of influence shows that one should not only analyze the series in levels, but also the residuals of the series are important to predict its level.

The application of other nonlinear models capable of revealing the behavior of other processes, such as GARCH, EGARCH and TARCH models may be of importance for these studies, as well as other types of control charts such as, Exponentially Weighted Moving Average - EWMA and Cumulative Sum - CuSum Charts.

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