Complex Variables for Strategy: From Bombelli's Imaginary Number, Steinmetz's Phasor Domain to Sunzi's Art of War

R. Iembo
Faculty of Engineering
University of Calabria
Cosenza, Italy
r.iembo@unical.it

Chong Ming Lin
Chang Chun University, China
Sunnyvale, California, 94087, USA
lcm200001@yahoo.com

ABSTRACT

Rafael Bombelli was born in Bologna,Italy, on 1526. He was trained and worked as an engineer-architect until died on 1573 without much recognition until **hundred years later by the math society as a great Italian mathematician**, and there is a lunar crater named Bombelli for his contributions to imaginary and complex numbers .

Bombelli is the author of a treatise on algebra and is **a central figure in the understanding of imaginary numbers**. His mathematical achievement was never fully appreciated during his life time, but his failure to repair the Ponte Santa Maria 1561 attempt, a bridge in Rome, was probably fully recognized by his comtemporaries, until hundred years later the great mathematician Leibniz (1646-1716) recognized his great contribution to the finding and operation of complex numbers and hailed him an "outstanding master of the **analytical art** !"

In ***Algebra*** 1569, Bombelli solved equations, using the method of del Ferro/Tartaglia, he introduced +***i*** and -***i*** and described how they both worked in ***Algebra***. But it's interesting to note that even Bombelli himself had never realized his own great contribution to math, since he still referred to complex numbers as useless, not to mention his contemporaries. Nonetheless, Bombelli's ***Algebra*** was finally widely read and respected by the math giants such as Leibniz used it to study cubic equations and Leonhard Euler quoted from it in his text, ***Algebra***.

Bombelli's contribution to math, especially his understanding about real numbers can result from the operations of complex numbers, led to the breakthrough in calculation of alternating current and voltage with R, C & L (Resistance, Capacitance & Inductance) as the three variables to represent the operation of any electrical devices or power lines. Looking back at the development of modern societies supported by high-tech electronic devices and energies supplied from alternating current based powerlines, we should even give higher praise to Bombelli's finding and development of imaginary numbers and operation of complex variables in the first place.

With understanding of the invention and finding of imaginary numbers and operation of complex variables, the authors would like to discuss how the strategies outlined by the greatest strategist **Sunzi**, who lived about 2500 years ago in China, who wrote the classical military text, "***Art of War***", can be in line of the concept and operation of complex variables. In this paper, the authors will combine the understanding of imaginary numbers by an Italian mathematician and the studies of a Chinese engineer and Sunzi strategy researcher, to show that Sunzi's military strategy based on ***I-Ching's*** philosophy with Yin & Yang as variables, can related to the Real & Imaginary numbers, and their operations and transitions from real to imaginary field in strategical thinking and business planning in daily life as well.

1. INTRODUCTION: IMAGINARY NUMBERS AND NEGATIVE NUMBERS

The solution of the equations of third and fourth degree were perhaps the greatest contribution given to algebra from the time of the Babylonians who taught "to complete the square" solving the equation of second degree. No other discovery stimulated so the development of algebra as the discovery disclosed in ***Ars Magna***.

The solutions of the equations of third and fourth degree were not the result of practical reflections, nor they had practical aims. The most important consequence of the discoverys published in ***Ars Magna*** was the potent spur which they gave to the algebraical searches in different directions.
A result of the solution of the equation of third degree was a new " kind" of numbers.

At the time of Cardano the irrational numbers were by now admitted, though they had not became a solid foundation because they could be approximated by rational numbers. The negative numbers could not be approximated by positive numbers but the notion of direction on a line made them convincing.
Cardano used them but he called them "numeri ficti".

If an algebraist wanted to deny the existence of irrational or negative numbers he could say, as the ancient Greeks, that the equations $x^2=2$ and $x+2=0$ had not solutions. Similarly the algebraists had avoided the imaginary numbers saying that an equation like $x^2+1=0$ had not soultions. They did not need square roots of negative numbers. The solution of the equation of third degree changed radically the situation.

When the three roots of the equation of third degree were real and different from zero, the Tartaglia and Cardano formula gives square roots of negative numbers. The results had to be a real number but it could not be obtained without understanding the imaginary numbers. At this point another great italian algebraist Rafael Bombelli had an idea which he called "an absurd idea" because he seemed that such idea was based on sophistical considerations.

2. "AN ABSURD IDEA": Rafael Bombelli

Rafael Bombelli (1526–1573), was the author of a treatise on algebra and was a central figure in the understanding of imaginary numbers.

The career of Italian algebraist Rafael Bombelli helped bridge the late Renaissance and the early period of the Enlightenment. The last among many Italian mathematicians who contributed to a developing theory of equations, Bombelli became the first to conceive a consistent theory of imaginary numbers including rules for operations on complex numbers. His work, whose implications for complex numbers Bombelli never fully appreciated, won him admiration among future mathematical giants such as Gottfried Wilhelm von Leibniz (1646-1716) and Leonhard Euler (1707-1783).

Bombelli opted not to follow his father, Antonio, in the latter's profession as wool merchant. Instead, he became an engineer, and rather than attending university, Bombelli received his training as an apprentice to engineer-architect Pier Francesco Clementi of Corinaldo. He spent much of his career under the patronage of the Bishop of Melfi, Monsignor Alessandro Rufini, and was responsible for the draining of the Val di Chiana marshes (1551-60), as well as the unsuccessful 1561 attempt to repair the Ponte Santa Maria, a bridge in Rome.

Bombelli's work in mathematics began during the 1560s, first with the writing of ***Algebra***. The latter is particularly significant because it reintroduced scholars to the work of the ancient Greek mathematician Diophantus of Alexandria (3rd century A.D.) In addition to reproducing 143 problems of Diophantus, whose writings Bombelli had discovered in the library of the Vatican, the book also introduced a number of symbols in algebraic notation. Attempting to improve on the already established Cardano-Tartaglia formula, Bombelli set out to develop his own highly precise theory of imaginary numbers (e.g., the square root of a negative number). In the following, let's take a look to understand Bombelli' idea.

We know that the two radicands of the cubic roots which follow from the usual formula of the equation of third degree differ only for a sign. For example the solution of $x^3 = 15x+4$
by the formula, is

$$x = \sqrt[3]{2+\sqrt{-121}} + \sqrt[3]{2-\sqrt{-121}}$$

while for direct substitution we know that x=4 is the only positive root of the equation.

By his ingenious method Bombelli had showed the important role that the complex numbers would have had in the future; but in that moment his observation will not serve for solving the equations of third degree because Bombelli had to know preliminarly one of the roots. In such case the equation was already solved. So Bombelli method was inefficacious. This was the situation whenever the three roots were real; so this was called the "irreducible case". In his book ***Algebra*** Bombelli solved equations, using the method of del Ferro/Tartaglia, he introduced +***i*** and -***i*** and described how they both worked in Algebra.

Bombelli wrote his ***Algebra*** text in 1560, just a few years after Girolamo Cardano's ***Ars magna*** was published. Bombelli's ***Algebra*** was not published until 1572 (and only a partial edition), and it became a very important work for several reasons. First, it featured 143 problems found originally in Diophantus' Arithmetica. Bombelli had found a copy of Diophantus' book in the Vatican Library and until that time the ancient Greek's Diophantus' work was mainly ignored. Second, in ***Algebra***, Bombelli made notable contributions to improvements in algebraic notation. He introduced an index notation for denoting powers, which he referred by the term "dignita." Third, Bombelli also introduced in this work a new symbol to indicate the root of an entire expression. It was the predecessor of a modern bracket.

Bombelli never realized his great contribution, since he still referred to complex numbers as useless. Nonetheless, Bombelli's ***Algebra*** was widely read and respected. Later on, Leibniz used it to study cubic equations and Leonhard Euler quoted from it in his text, ***Algebra*** .

3. "A SIMPLE PROBLEM IN ALGEBRA" : Charles P. Steinmetz's "*Phasor Algebra*"

Some historians like to say that America's rising to world power was due to its huge land and opening to two major oceans. But for people who regarded science and engineering as the major reasons for America's quickly move to the front of the world pwoers is due to Thomas Edison and Charles Steinmetz's inventions on electrical devices and utilization of AC power through long distance to support this huge land for its industrial development and commercial activities. In this regard, Charles Steinmetz's invention and proof of "Phasor Algegra" in 1893, had revolutionized AC circuit analysis by reducing it to a problem of algebra, completely eliminating the need for calculus, and allow any electrical engineer who knows high school algebra and trigonometry to calculate and analyze R,C and L based AC circuitries without using complicated calculus or approximation method in designing new systems. Since the calculation of complicated RCL (Resistance, Capacitance and Inductance) AC circuitries were becoming simple algebra or arithmetical steps, therefore Steinmetz proudly joked about his invention of using phasor algebra to analyze complicated AC circuits and systems as "a simple problem in algebra."

Although Steinmetz understood well about how to anaylze sinusoidal AC circuits that have real and imaginary components in real time operation, yet he treated those "religious thinking/belief" as "mental power" or "mental energy" , and gave it a name as "The Entity 'X'", which has no real energy involved, therefore, he wrote in ***Science and Religion***[1], "obvious that the alleged manifestations of spiritism must be fake or self-deception, since they are manifestations of energy. Entity "X," if it exists, certainly is not energy, and therefore could not manifest itself as such."

But from the phasor algebra that shows energy can be stored in capacitor and inductor, and when their relative phases shifts with time, they can have both real and imaginary values relative to the ohmic behavior resistors that always have positive real numbers in value. Therefore, in electrical system, real and imaginary values can be temporary and they can be changed with time, and in most of time, most of the electrical components have both real and imaginary values as function of time. When phasor quantity is added to the measurement, the value of any electrical component have two quantities, absolute real number magnitude and time varying phasor angle. Steinmetz had another famous quote: **"Money is a stupid measure of**

[1] Harpers Magazine for February, 1922.

achievement, but unfortunately it is the only universal measure we have." But in real world, we human beings do have another dimension to operate in addition to the material touchable world --- the mental world, or the "soft power/energy" vs. The tangible material world of "hard power/energy", that only have limited power in developing strategy to deal with the real world problems. In this regard, we have to go back to the ancient strategist Suntzu's classic book, ***The Art of War***, to see the complex ways with Yin and Yang considered, for making any effective strategy in dealing with human world.

4. COMPLEX WAYS FOR STRATEGY: Suntzu's "*Art Of War*"

Sunzi's "Art of War" was written 2500 years ago. His main contribution to the art of war is forming a strategical thinking system for military leaders to handle military operation under a philosophy of Yin & Yang derived from "***I-Ching***", with considerations from both regular cases and unexpected moves where the "real" and "imaginary" forces and/or values can be maneuvered to change into the other domain. As in Steinmetz's phasor algebra and phasor domain, in our real world that are always the tangible material part that we can see and can measure, and there are also the intangible mental and cultural world, that we can comprehend but cannot touch it and measure it with material means. Therefore, in Suntzu's ***Art of War'***s very first chapter, Suntzu wrote that the winning strategy in war is to convince your people to agree with you, your material preparation as well as your vision and credibility. Suntzu always advises his readers to consider battle and war by numbers and ratios, and yet he also reminds his readers that the unseen part, the environment, the thinking of your enemy, the changing direction, the momentum and the potential of military forces , that can change from imaginary part into realistic force is just as important to manipulate as the tangible material assets of soldiers and weapons. Looking at current war in Iraq and the war against terrorists of ideological based hatred, shall we say that we must treat the combination of the "hard power" and the "soft power" together, using them as a combined vector as in Steinmetz's phasor domain, and calculate the problems and opportunities in this complex world with the mind that calculate complex variables as complex variables, not as real numbers. Then we can handle the human related national and international affairs more effectively.

CONCLUSIONS

Knowledge management depend upon how we manage knowledge types. Design effective strategies to handle complex human related issues needs thinking of issues in complex variables as Steinmetz analyzes AC circuits and electrical system, and Suntzu's way of looking at battles with the real and the imaginary parts considered together. Many of the current international crisis are due to leaders of power states who ignored the human world is made of real and imaginary components, of soft and hard powers.

In looking back to the development of complex variable math and phasor domain and complex algebra applied to electrical systems, we should start to think about how we can benefit from reading Suntzu's Art of War and applies it to more effective strategies in handling human related issues, and to achieve lasting peace in this resource limited material world. Steinmetz had one notable quotation, that "**No man really becomes a fool until he stops asking questions."** That is exactly why the authors write this paper, to demonstrate the relation between math development and management strategy. We hope this paper will open a new field in understanding the importance of complex variable that can apply to daily events as well as to international relation with better strategical model in calculation and in thinking than mathematicians and strategists had ever thought before.

REFERENCES

[1] Carl B. Boyer, **Storia della Matematica**, Mondadori, Milano, 1980

[2] Jean Dieudonnè, **L'Arte dei Numeri**, Mondadori, Milano, 1989

[3] Morris Kline, **Mathematical Thought from Ancient to Modern Times**, Oxford University Press, New York, 1972

[4] Lin Chong Ming, **Table of Complementary Opposites,** 2004.7.25

[5] Floyd Miller, **The electrical genius of Liberty Hall: Charles Proteus Steinmetz** , McGraw-Hill, 1962

[6] Charles Proteus Steinmetz, **Lectures on Electrical Engineering,** Vol. I (Dover Phoenix Editions) , Dover Publications (February 20, 2003).

[7] David Eugene Smith, **A Source Book in Mathematics**, Dover Publications, New York, 1959

FIGURE & TABLE

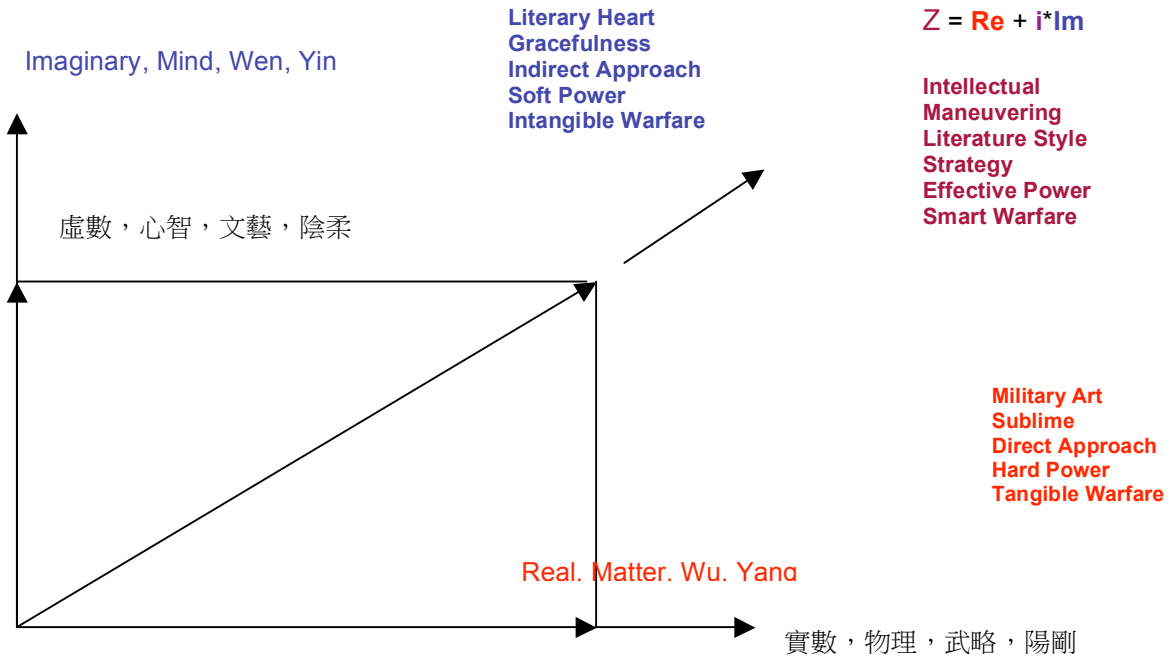

Fig.1. Complex Complimentary Variables in Complex Real World

TABLE 1. Table of Complementary Opposites

TYPE A	TYPE (A)'	COMMENTS
BASIC COMPLEMENTARY OPPOSITES: "Yin" & "Yang"		
Yang 陽	Yin 陰	I-Ching: Yin & Yang unified to form the Tao
Philosophy		
Matter	Mind	"Mind-body problem" may live longer than Materialists like to see under Cognition Theory
Occupation	Emptiness	Laotzu《老子》
Sudden 頓悟	Gradual 漸悟	Zen Enlightenment
Math		
Re: Real Number	Im: Imaginary Number	Complex Variable Z = Re + i*Im
Physics		
Particle	Wave	Light Property
Human Activities Under "Wen" & "Wu"		
Wu 武略	Wen 文 "心"	Concept and Skill in Basic Human Activities
Military Art	Literary Heart	Intellectual Maneuvering
Direct Approach	Indirect Approach	Strategy = D.A. + I.A.
Hard Power	Soft Power	Power = Hard + Soft P.
Sublime	Gracefulness	Literature Style = S.+G.
Warfare		
Tangible Warfare: Army, Navy, Air Force...	Intangible Warfare: Media, Legal, Psycho...	Smart Warfare = Tangible + Intangible W.
Missile & Bomb	Info & Cyber	Hi-Tech weapons
Culture Study		
Science	Liberal Arts	"The Two Cultures"
Technology & Business		
Hardware	Software	Computer Technology
Manufacturing	Design	Engineering
Innovation	Execution	Hi-Tech Company Operation