Measuring the Refractive Index of Liquid and Gas by Mie Scattering

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ABSTRACT

In this study I create a new method to measure the refractive index of liquid and gas by using Mie theory. The method is initiative though someone has used Mie scattering to measure the radius of small particles. This technique has the advantage of large range of measurement and tiny volume. I also provide two cases of the refractive index measurement devise based on this method in this paper.

Keywords: The refractive index, Mie theory, range of measurement, tiny volume and light distribution.

1. INTRODUCTION

The refractive index of liquid and gas is very important for industrial use. As the refractive index is one of the key indexes of liquid and gas, it is measured frequently in the laboratory when scientific researchers are testing or creating different kinds of solutions and gases.

Advancements in diffraction and interference technology have already made it possible for the accurate measurement of the refractive index. However, the scattering technology has not been utilized to measure this kind of index. Mie theory has been used to measure the refractive index of the particle itself [1], but it has been seldom used to find out the index of the liquid or air where the particles immersed in. So I employ the Mie theory which has been brought forward hundreds years ago to make it more useful in the liquid and gas index measurement. Besides, my design has little cost of fund and room, also the cost of the liquid and gas when they are expensive as this technology needs little sample to be test.

2. BACKGROUND

There are many other ways to measure the refractive index of the medium. Basically, they can be classified in three kinds [2]. I will introduce each of them in this section and present the basic knowledge of Mie theory later on.

Background Technology

(1)Geometric optical technology: This technology has critical angle method [3], imaging method, spectrometer method [4] and Abbe refractometer method. The main idea of the technology is to measure the angle caused by refraction or reflection and calculate the refractive index as the angle is relate to the refractive index of the medium which the light travel through. In this kind of technology, spectrometer has better resolving power but Abbe refractometer method is weak in this aspect.

(2)Wave optical technology: This technology has interference method and polarization method. The interference method based on the light length difference after the light travels through the medium, and people can judge the refractive index via counting the interference stripe. Polarization method is based on the principle that the polarization state will change when the light refract at the surface of the medium.

Nowadays many measurement systems use CCD to count interference stripe. For example, Shiqun Hua [5] used CCD to manipulate stripe based on their CCD automatic test system.

(3)Fiber optical sensors technology: This kind of technology is the most frequently used method nowadays for its high sensitiveness and the ease of operation. The most representative method to measure the refractive index is the fiber grating method [6] and the optical fiber refractive sensor based on surface plasmon resonance [7].

Fiber grating can be divided into Bragg grating and long-periodic grating. The fiber grating is very sensitive to the change of the refractive index outside. The external condition change will affect the radius and the refractive index of the fiber core and the clad. Thus, using the relations between the refractive index change outside and the amount of movement of the resonance hump, we can measure the refractive index outside accurately.

The most popular technique used to measure the refractive index is the fiber refractive sensor based on surface plasmon resonance. It can measure the refractive index of liquid with a high precision [8-9]. A plasma wave resonates effect is a physical chemistry phenomenon happened on the interface between the metal and electrolytes, and it is very sensitive to the change of the refractive index of the medium surrounding them.

The Basic Knowledge of Mie Theory

People will draw into the dimensionless diameter index $\alpha=m_1\pi d/\lambda$, where m_1 is the refractive index of the medium surrounding the particle, λ is the wavelength of the incident light in vacuum, and d is the diameter of the particle.

Considering the condition of polarization, let the incident light go toward z direction, and electric vector goes toward x direction. The distance between the particle and the observation point of the scattering light is r. The scattering angle is represented by θ. The plan constituted by z axle and observation point is called scattering plane. The angle between the oscillating plane of the incident light and the scattering plane is φ. Now I_r represents the intensity of the light which vibrates perpendicular to the scattering plane; I_l represents the intensity of the light which vibrates parallel to the scattering plane; I_s represents the total scattering light intensity. Their expression is as follows [10]:

$$I_r = \frac{\lambda^2}{4\pi^2 r^2}|s_1(\theta)|^2 I_0 \sin^2\varphi = \frac{\lambda^2}{4\pi^2 r^2} i_1(\theta) I_0 \sin^2\varphi \tag{1}$$

$$I_l = \frac{\lambda^2}{4\pi^2 r^2}|s_2(\theta)|^2 I_0 sin^2\varphi = \frac{\lambda^2}{4\pi^2 r^2} i_2(\theta) \mathrm{I}_0 \sin^2\varphi \tag{2}$$

$$I_0 = \frac{\lambda^2}{4\pi^2 r^2}\left[i_1(\theta)\sin^2\varphi + i_2(\theta)\cos^2\varphi\right] \tag{3}$$

For spherical particles, the scattering angle, the relative refractive index $m=m_2/m_1$ (m_2 is the refractive index of the particle itself) and the dimensionless index α which represents the particle radius can influent the intensity function $i_1(\theta)$, $i_2(\theta)$ and amplitude function $s_1(\theta)$, $s_2(\theta)$. But the azimuth angle φ has no effect on them. The infinite series constituted by Legendre function and Bessel function make up of the amplitude function.

$$s_1(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)}(a_n\pi_n + b_n\tau_n) \tag{4}$$

$$s_2(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)}(a_n\tau_n + b_n\pi_n) \tag{5}$$

a_n and b_n are called Mie scattering index, which is the function constituted by dimensionless index α and refractive index m; But τ_n and π_n is relative to scattering angle θ.

$$a_n = \frac{\psi_n(\alpha)\psi_n'(m\alpha) - m\psi_n'(\alpha)\psi_n(m\alpha)}{\xi_n(\alpha)\psi_n'(m\alpha) - m\xi_n'(\alpha)\psi_n(m\alpha)} \tag{6}$$

$$b_n = \frac{m\psi_n(\alpha)\psi_n'(m\alpha) - \psi_n'(\alpha)\psi_n(m\alpha)}{m\xi_n(\alpha)\psi_n'(m\alpha) - \xi_n'(\alpha)\psi_n(m\alpha)} \tag{7}$$

$$\pi_n = \frac{\mathrm{P}_n^{(1)}(\cos\theta)}{\sin\theta} = \frac{\mathrm{dP_n}(\cos\theta)}{d\cos\theta} \tag{8}$$

$$\tau_n = \frac{\mathrm{dP}_n^{(1)}(\cos\theta)}{d\theta} \tag{9}$$

$\xi_\mathrm{n}(z)$ and $\Psi_\mathrm{n}(z)$ are Ricatti-Bessel functions, and they belong to the functions of the second Hankel function and semi-integral order Bessel function.

$$\xi_n(\mathrm{z}) = \left(\frac{\pi z}{2}\right)^{\frac{1}{2}} H_{n+\frac{1}{2}}^{(2)}(z) \tag{10}$$

$$\psi_n(\mathrm{z}) = \left(\frac{\pi z}{2}\right)^{\frac{1}{2}} J_{n+\frac{1}{2}}(z) \tag{11}$$

$P^{(1)}{}_\mathrm{n}(\cos\theta)$ and $P_\mathrm{n}(\cos\theta)$ are the one-order associate Legendre function and Legendre function based on $\cos\theta$.

When the incident light is the natural light, the scattering light is partial polarized. Like former formulas, the scattering intensity is:

$$I_r = \frac{\lambda^2}{8\pi^2 r^2}\mathrm{i}_1(\theta) I_0 \tag{12}$$

$$I_l = \frac{\lambda^2}{8\pi^2 \mathrm{r}^2}\mathrm{i}_2(\theta) I_0 \tag{13}$$

Thus we can get the polarization degree P as Eq. (14) and the total scattering intensity I_s as Eq. (15):

$$P = \frac{i_1(\theta) - i_2(\theta)}{i_1(\theta) + i_2(\theta)} \tag{14}$$

$$I_s = \frac{\lambda^2}{8\pi^2 r^2}\left[i_1(\theta) + i_2(\theta)\right] I_0 \tag{15}$$

For polarized light and the natural light we have:

$$k_{sca} = \frac{C_{sca}}{\pi a^2} = \frac{2}{a^2}\sum_{n=1}^{\infty}(2n+1)(|a_n|^2 + |b_n|^2) \tag{16}$$

$$k_{ext} = \frac{4}{a^2}\mathrm{Re}\left[s(\theta)\right] = \frac{2}{a^2}\sum_{n=1}^{\infty}(2n+1)\mathrm{Re}(a_n + b_n) \tag{17}$$

$$k_{abs} = k_{ext} - k_{sca} \tag{18}$$

3. FEASIBILITY ANALYSIS

To measure the refractive index of liquid and gas, we need scatterer which is illuminated by the incident light. The scatterer may be several particles with the radius between 10nm to 10μm. I will choose the case of one small particle in this paper. The scatterer is located in a cavity with two tubes used as entrance and exit for the liquid and gas. The tubes are curving or zigzag to make sure that the light outside will not come in the cavity. There is one or more light intensity detectors distributed surround the scatterer to detect the scattering light intensity in the setting direction. The inwall of the cavity is paint with material which has high absorptivity of the incident light.

Now let's concentrate on the measurement of the refractive index of liquids, and the measurement of the refractive index of gases is similar to it. As the range of the refractive index of liquids spreads lager than the gases', the test on liquids is more representative. The refractive indexes of gases vary from 1.000132 to 1.001711, and the indexes of the liquids we have ever got are distributed between 1.31 and 2.10. The refractive indexes of the most familiar liquids in daily life are distributed between 1.33 and 1.36. We need to set the radius of the scatterer and the location of the light intensity detectors, so when the refractive index changes little, the variability of the light intensity which detectors can detect is distinguishable for detectors.

Based on the functions in Section 2.2, I compile a program to calculate the scattering light distribution after the incident light travel through a spherical particle [11]. Ground on this, A program is compiled by Matlab in order to set the location of the detector and the radius of the particle in the case that only one small particle is used. The program is attached in the appendix.

In the program, the wavelength of the incident light is set at 1.31μm, and the incident light is same each time. I divide the whole work into several groups. In each group, I set the radius of the particle at one specific value. Then I locate detectors at the position where the scatter angle is 0.1π, 0.2π, 0.3π, 0.4π, 0.5π, 0.6π, 0.7π, 0.8π, 0.9π and 1.0π. After that, I change the refractive index of the medium surround the particle from 1.33 to 1.36 with the separation of 0.005. Finally, I collect the variance of the light intensity detected by the detector at each scatter angle. The larger the variance, the easier detectors can distinguish the change of the refractive index, which means the survey meter is more sensitive.

After collect the data of the first group, I change the radius of the particle and collect the data of the second group like the first step. The simulation data after running the program is listed as below. The variance with specific radius and the scattering angle is listed in Table 1.

From the statistical table we can summarize that generally, the variance of the scattering light at forward direction is increasing along with the increasing of the radius of the particle. This is because when the radius is increasing, the scattering light will concentrate on the forward direction. Thus, the base is larger, and the change of the refractive index will bring larger difference between the light intensity.

The conclusion that the scattering light concentrate on the forward direction along with the increasing of the radius is demonstrated by running the program calculating the light distribution after Mie scattering. For instance, Fig.1 is the statistical chart of the ratio of the backward energy of Mie scattering under the condition that the relative refractive index m ($n_{particle}/n_{surrounding\ medium}$) equal to 0.9.

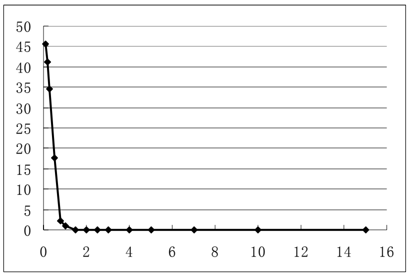


Fig.1. The ration of the backward energy changes along with diameter when m=0.9

So in general, choosing larger spherical particle will enhance the sensitivity of the refractive survey meter. Furthermore, locating the light intensity detector at the forward direction will make it detect more energy when the refractive index of the medium changes. However, if enlarge the radius without any limitation, the spherical particle will be too large and range out of the confine of Mie theory.

For example, If only one light intensity detector is used in the cavity, setting the radius at 4.0μm, and the light intensity detector is located at the scattering angle of 0.1π; if two light intensity detectors are used, setting the radius at 3.0μm, and light intensity detectors are located at the scattering angle of 0.1π and 0.2π separately.

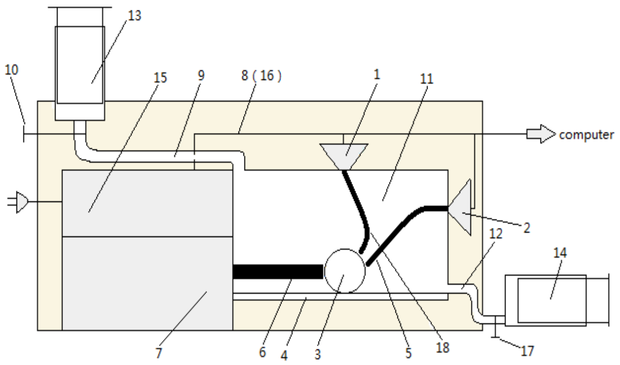
Table 1 The variance with specific radius and the scattering angle

Radius/ Angle	**0.1π**	**0.2π**	**0.3π**	**0.4π**	**0.5π**	**0.6π**	**0.7π**	**0.8π**	**0.9π**	**1.0π**
0.3μm ($\times10^{-6}$)	0.1988	0.1509	0.0739	0.0130	0.0015	0.0215	0.0407	0.0477	0.0474	0.1577
0.4μm ($\times10^{-5}$)	0.1300	0.0692	0.0168	0.0000	0.0068	0.0143	0.0144	0.0105	0.0070	0.0616
0.5μm ($\times10^{-6}$)	0.0836	0.0251	0.0078	0.0806	0.0901	0.0265	0.0014	0.0507	0.1261	0.0522
0.6μm ($\times10^{-6}$)	0.0000	0.0653	0.1606	0.0742	0.0004	0.0237	0.0420	0.0388	0.0339	0.0434
0.7μm ($\times10^{-6}$)	0.9380	0.0740	0.2950	0.0072	0.0439	0.0163	0.0042	0.0261	0.0237	0.0007
0.8μm ($\times10^{-6}$)	0.5732	0.5277	0.1963	0.0405	0.0312	0.0025	0.0065	0.0028	0.0430	0.0088
0.9μm ($\times10^{-5}$)	0.0388	0.1295	0.0016	0.0095	0.0001	0.0002	0.0014	0.0021	0.0012	0.0001
1.0μm ($\times10^{-5}$)	0.0703	0.1808	0.0079	0.0053	0.0003	0.0015	0.0010	0.0004	0.0013	0.0000
1.1μm ($\times10^{-5}$)	0.0297	0.1634	0.0333	0.0000	0.0005	0.0006	0.0004	0.0000	0.0012	0.0001
1.2μm ($\times10^{-6}$)	0.0022	0.9268	0.4207	0.0051	0.0301	0.0019	0.0015	0.0201	0.0025	0.0001
1.3μm ($\times10^{-6}$)	0.0951	0.1346	0.2388	0.0038	0.0059	0.0000	0.0142	0.0000	0.0074	0.0002
1.5μm ($\times10^{-5}$)	0.3369	0.1920	0.0006	0.0072	0.0000	0.0006	0.0001	0.0019	0.0003	0.0000
1.8μm ($\times10^{-4}$)	0.1675	0.0999	0.0038	0.0000	0.0000	0.0002	0.0000	0.0001	0.0001	0.0000
2.0μm ($\times10^{-4}$)	0.1841	0.0715	0.0087	0.0021	0.0003	0.0000	0.0003	0.0001	0.0000	0.0000
3.0μm ($\times10^{-4}$)	0.7356	0.6589	0.0049	0.0003	0.0000	0.0000	0.0001	0.0001	0.0003	0.0000
4.0μm ($\times10^{-4}$)	0.4223	0.1461	0.0235	0.0026	0.0000	0.0020	0.0003	0.0000	0.0000	0.0000
5.0μm ($\times10^{-3}$)	0.5684	0.3591	0.0254	0.0000	0.0002	0.0001	0.0001	0.0002	0.0003	0.0000

4. EXAMPLES OF THE REFRACTIVE SRUVEY METER

The light source output laser at 1.31μm. The light travel through the incident fiber and the order of the core diameter of incident fiber is 1μm. There are narrow gaps between the ends of the fibers and the particle, and the order of the width of the gaps is 1nm. The particle is fixed on the fixed tray by cauterization or pasting and its radius is 3μm. The scattering light is detected by two detected fibers at the scattering angle of 0.1π and 0.2π. The test preparation is contained in the vessel and will inpour into the measurement cavity when the plunger is open. After measurement, open the plunger of the vessel containing outflow liquid (gas) and the testing material will be extracted. The measurement cavity need to be washed by distilled water each time after measurement. In addition, each time before measurement, close the upper plunger and open the lower one, extract the air in the measurement cavity to approximative vacuum.

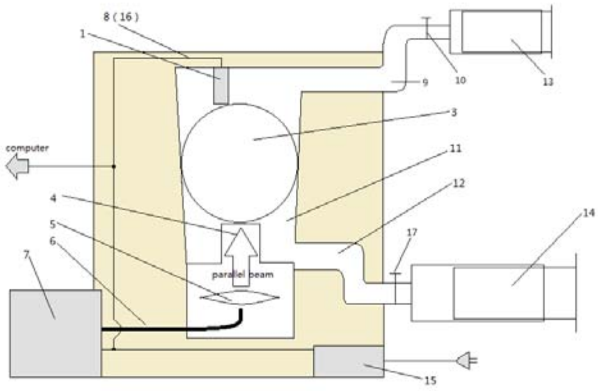
Case 1:



1, 2—light intensity detector; 3—particle; 4—fixed tray; 5, 18—detected fiber; 6—incident fiber; 7—light source; 8—data wire; 9—tube for infused liquid(gas); 10, 17—plunger; 11—measurement cavity; 13—vessel contain infused liquid(gas); 14—vessel contain outflow liquid(gas); 15—energy source; 16—power line

Fig.2. The schematic diagram of Case 1

Case 2:



1—light intensity detector; 4—incident light emitting surface; 5—lens; 6—incident fiber; 7—light source; 8—data wire; 9—tube for infused liquid(gas); 10, 17—plunger; 11—measurement cavity; 13—vessel contain infused liquid(gas); 14—vessel contain outflow liquid(gas); 15—energy source; 16—power line

Fig.3. The schematic diagram of Case 2

Before testing the unknown refractive index of medium, we need to use the liquids and gas which the refractive indexes are known to test the relation schema of the detected light intensity and the refractive index of medium. In the case of this example, a three-dimensional schema should be made. Two axises represent the light intensity detected by two detectors, and the third axis represents the refractive index of the medium. Using mediums which the refractive indexes are known, we can depict the relation schema by computer. When the unknown refractive index is measured, the energy detected by the two detectors will set the value of the first two axises of the relation schema and we can easily find out the value of the refractive index.

The difference between the Case 1 and Case 2 is that only one light intensity detector is used here. The particle has the radius of 4μm and is fixed by four slopes of the measurement cavity. The lens is used to assemble the incident light into parallel beam. The other elements act similarly as the first case.

5. CONCLUSION

The refractive index meter based on the Mie theory is initiative and has many advantages. It has a large range of measurement and can detect the refractive index of liquids and gases. Its volume is little and can be integrated on other devices. The operation is simple and the cost is low. The scattering light distribution changes with the change of the refractive index of the medium surround the scatterer, thus detect the change of light distribution will lead us to find out the refractive index of the medium. Optimize the size of the particle and the location of the light intensity detector will increase the sensitivity of the survey meter. Two examples are given to illustrate the technique.

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