The potential of e-learning platforms to communicate mathematics

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ABSTRACT

This paper presents some ideas for the design of online activities for mathematics blended courses. The focus is on the integration of technology and research in the field of mathematics education, with special concern for language and semiotics.
In the section 'Background' we give:

- an overview of the outcomes of research that underline the complexity of educational processes, and in particular the need for taking into account not just cognitive, but also meta-cognitive and non-cognitive aspects;
- a framework for dealing with language and representations in order to effectively interpret students' behaviors.

In the section 'The potential of e-learning' we show how some of the activities provided by a standard e-learning platform can help to implement some of the ideas presented in the 'Background' section.
In the section 'Teaching and learning opportunities' we show examples of teaching activities which fulfil some of the requirements sketched in the previous sections and apply some of the ideas and methods discussed there.
The section "Future trends and conclusions" includes some discussion of the opportunities for future research.
In all the examples described in this paper we mainly refer to Moodle (see Moodle documentation).

Keywords: e-learning, blended course, mathematics education, constructivism, semiotics, linguistics, pragmatics, register.

BACKGROUND

Mathematics Education, Technology and Research

Nowadays information and communication technology (ICT) is not closely related to any theoretical framework in mathematics education. In the past, on the contrary, sometimes it was naively associated to some cognitive framework or even to some way of interpreting mathematics. This might explain the relatively poor role played by ICT in most studies in the psychology of mathematics education.
Needless to say, the use of ICT is far from being a simple matter but wants the development of comprehensive teaching units and plenty of research to make the most of the opportunities provided and to avoid any possible shortcoming.
A range of studies on mathematics education, on the other hand, has highlighted the complexity of teaching and learning processes. This means that oversimplified frameworks, including the belief that just the addition of technology to standard practices could provide considerable improvements of the outcomes are utterly inadequate.
Above all, mathematics education has to take into account that learning outcomes are influenced by factors belonging to at least three separate levels:

- the non-cognitive level, which refers to beliefs, emotions and attitudes, and all affective aspects, which are most often critical in shaping learners' decisions and performances;
- the meta-cognitive level, which refers to learners' management of their own processes;
- the cognitive level, which refers to the acquisition of the characteristic ideas and methods of the discipline, with special care for to the obstacles recognized by research and practice.

As we will see below, ICT (and thus e-learning platforms) can be relevant at each level, including the non-cognitive one. As a matter of fact, it can deeply affect learners' beliefs, emotions and attitudes towards mathematics, and moreover it is itself the object of deep-rooted beliefs and can influence the non-cognitive aspects.
So any investigation combining ICT and mathematics education needs to consider non-cognitive factors concerning both technology and mathematics.

Constructive Methods

In mathematics education the constructivist perspective plays a major role. In the past ICT has been regarded as conflicting with such methods by a good share of researchers in mathematics education. More recently technology has proved able to support a wide range of teaching methods, although this has not still been acknowledged by all researchers. Most likely some scholars would adopt a more restricted view of constructivism and would regard some computer environments and, more generally, some ways of using ICT as inconsistent with the constructivist perspective. For example, graphing a function, (defined by a symbolic expression) by means of the facilities of some Computer Algebra System might be regarded as non-constructive as some steps of the process are fully concealed to the learner, whereas programs explicitly computing the coordinates of a finite set of points of the graph of the function might be regarded as more suitable for a truly constructive approach. Although we understand some of the concerns of the supporters of the restricted view, in this section we are adopting an inclusive definition of constructivism. An e-learning platform allows the learners to develop new knowledge as they interact with the environment. Within an e-learning platform the learner can freely use a range of modules to construct his or her knowledge. Modules allowing some feedback, such as Moodle's 'lesson' or 'quiz' are specially relevant from this perspective.

Cooperative Learning

E-learning platforms generally provide a number of activities involving peer interactions or interactions between learners and tutors. Modules such as Moodle's 'workshop', 'wiki' or 'task' are generally suitable for designing activities of this kind. In the paper we describe some experiences with a 'workshop' module at undergraduate level. From the viewpoint of the theory of mathematics education, all of these activities can be framed within the so-called socio-cultural (or 'discursive') approach. For more information see Kieran *et al.* (2001).

It is widely acknowledged that the cognitive processes induced by talking, discussing and explaining to others the concepts to be learnt promote deeper level and higher-order thinking (Johnson & Johnson, 1987). In this framework we want to underline peer learning (Boud et al., 1999), which is meant as the use of teaching and learning strategies inducing students to learn with and from each other without the close intervention of a teacher. It includes peer tutoring and peer mentoring. When the students in a group act as both teachers and learners we talk about reciprocal peer learning. This may incorporate self and peer assessment whereby students actively develop criteria for assessment. Falchikov (2001) analysed the various peer tutoring techniques and the benefits linked to each of them. She found evidence of some improvement in comprehension, memory for lecture content, performance and facilitation in encoding and retrieval of material given by Guided Reciprocal Peer Questioning.

Language And Representations

The potential of information and communication technology as regards semiotic or linguistic issues is largely underestimated. Language is growing one of the most relevant issues for research on mathematics education. From the one hand, classes including students from different linguistic groups pose new teaching problems. On the other hand, at any level, including undergraduate, a large share of students' failures can be ascribed to linguistic issues. An increasing number of students, for example, seemingly cannot properly understand a written verbal text even if it is short and simple. A detailed investigation of language-related troubles is beyond the scope of this chapter. More details on this topic have been provided by Ferrari (2002, 2004). In this section we are going to focus on two aspects: R. Duval's (2005) investigation of semiotic representation systems and the pragmatic interpretation of mathematical language.

Coordination of Semiotic Systems: Duval's Theory of Semiotic Representation Systems provides a new insight on the role of semiosis in learning. Algebraic symbol notation, verbal language, Cartesian graphs and geometrical figures are examples of semiotic representation systems. The main activities described by Duval are:

- The construction of a representation within a semiotic system, such as writing a text or a formula or drawing a figure.
- The treatment of representations within a semiotic system, such as summarizing a verbal text, computing the derivative of a function given as a symbolic expression or transforming a geometrical figure.
- The conversion of representations from a semiotic system to another, such as verbally describing a figure, or writing a formula to represent the data of a word problem, or drawing a figure satisfying some condition verbally expressed, or building a table of numerical values extracted by some formula, or verbally describing the solving strategy of a problem.

Duval often refers to semiotic representation systems as 'registers'. We prefer to employ 'register' to denote a use-oriented linguistic variety, according to the definition widely accepted in the field of linguistics. According to Duval, the main goal of education as far as semiotics is concerned is what he names 'coordination of semiotic systems', which is the ability at using multiple representations of the same 'object' and moving quickly from one to another.

A problem involving real functions, for example, can be appropriately dealt with by the coordination of the verbal description of the function, its symbolic representation as an equation, its Cartesian graph and a table of values it assumes. The coordination of semiotic systems might improve both understanding and problem solving skills. From the one hand students who can coordinate semiotic systems are allowed to distinguish a concept from its representation (which usually proves much harder, if one can deal with one representation only), from the other hand, they can adopt the best strategies provided by each representation (for example, symbolic computation of the derivative of a function or visual search for a tangent on the graph). The same remarks hold for other subjects like rational numbers, which can be represented as fractions or as decimals. Decimal representations are more suitable in order to calculate sums and to compare number size, fractional representations are more suitable in order to calculate products and in general to carry out exact calculations.

ICT provides plenty of opportunities to use multiple representation. An e-learning platform can suggest a number of activities appropriate to the goal of achieving the coordination of semiotic systems. A quiz item, for example, might involve verbal texts, formulas and graphs. Let's see a sample of typical items that can be inserted in a quiz, numerical answer question and multiple choice question.

Consider the function f defined on reals by the equation

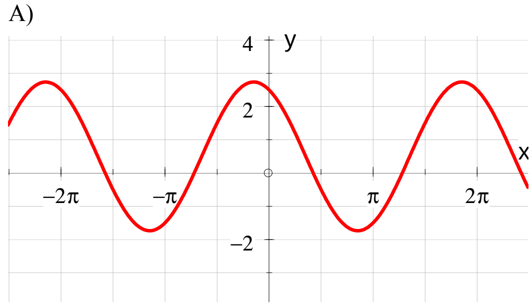
$$f(x) = 2\cos x - \sin x$$

a) Compute $f'(x)$.

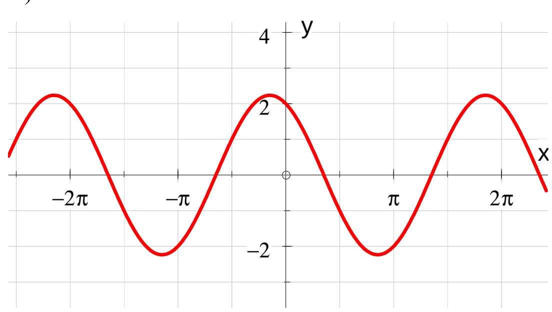
b) Compute $f'(0)$

c) Mark at least three of the following graphs that do not correspond to f.

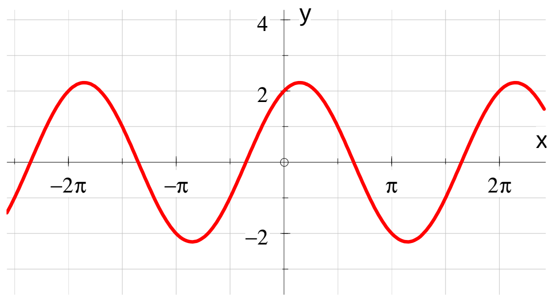
A)



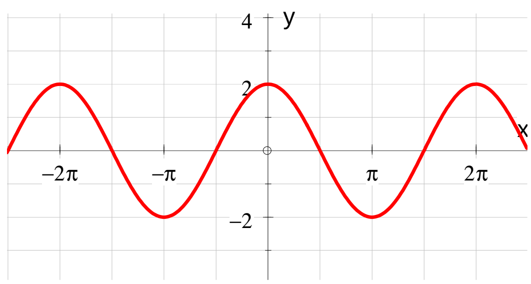
B)



C)



D)



Question a) can be implemented as a short answer question, question b) as a numerical answer question, question c) as a multiple choice question. Problems of this time require students to read all the kinds of representations (verbal text, graph, formula), to recognize properties of each of them and to combine the informations they can extract from each of them (e.g., from the symbolic expression of the derivative and the shape of the graph).

Pragmatics and Mathematical Language: Recently various frameworks have been proposed that underline the role of languages in the learning of mathematics. For example, Sfard (2000) interprets thinking as communication and regards languages not just as carriers of pre-existing meanings, but as builders of the meanings themselves. So, under this perspective, language heavily influences thinking. On the other hand, there is evidence that a good share of students' troubles in mathematics, at any school level, including undergraduates, can be ascribed to the improper use of verbal language. More precisely, as shown by Ferrari (2004), students often produce or interpret mathematical texts according to linguistic patterns appropriate to everyday-life contexts rather than to mathematical ones. The difference is not just a matter of vocabulary, grammar or symbols, but it heavily involves the organization of verbal texts, their functions and relationships with the context they are produced within.

Under these assumptions, a pragmatic perspective has proven suitable to provide tools to interpret students' behaviors and to design appropriate teaching units. A fine survey of the field of pragmatics has been provided by Leech (1983). In particular the functional linguistics approach, developed mainly by Halliday (1985), has provided a framework appropriate even from the epistemological perspective. Adopting these approaches means focusing on language use rather than on grammar, and regarding the interpretation of a text as a cooperative enterprise which involves not only vocabulary and grammar, but also the so called encyclopedia, i.e. the knowledge of the learner on the subject matter as well as on the world.. An e-learning platform provides plenty of opportunities for planning activities compatible with a pragmatic perspective. It is specially suitable for planning activities aimed at improving linguistic competence, including competence in verbal language, as it allows the authors to design a wide range of communication situations and to devise tasks forcing students to use more refined linguistic resources. An application of these ideas to advanced mathematics has been presented by Ferrari (2004). All of the activities described in the above paragraph on cooperative learning involve plenty of exchanges relevant from this perspective.

TEACHING AND LEARNING OPPORTUNITIES

Self-evaluation

Most of e-learning platforms provide the opportunity of designing sets of questions with automatic evaluation of the answers. The admissible formats for the items include *multiple choice, true/false, matching, fill-in, cloze-procedure, short answer, numerical answer*. Apart from short answer and numerical answer items, the other formats only require the learners to select their answer out of a prearranged set, and not to construct the answer themselves. This might be a critical issue. Items can be designed according to different criteria: they could be focused on one subject only, or on a whole course. In general, correct answers equipped with some comment are made available to students as soon as they have submitted their ones. Resources of this kind provide plenty of teaching opportunities, and some risk too. The item developers have to make the most of the benefits, exploiting the opportunities as much as possible, and to reduce the risks. This might make the development of the items a very troublesome business.

Students might use the sets of questions individually or in groups, to get immediate feedback about some aspects of their learning. This may greatly affect not just their knowledge, but their confidence too (the so called sense of autoefficacy). The opportunity of trying and making mistakes without the judgment of another human being may help some students to grow more confident and to develop a more positive attitude towards their products. Students could even use sets of questions as a means to learn: the interaction with the resource could be used to add some piece of knowledge. Using resources of this kind might prove somewhat risky, as some kinds of items might prove harder to develop and implement than others, which might imply that they are chosen less frequently, notwithstanding their effectiveness. For example, currently in most platforms is much easier and faster to insert word questions with little symbolic expressions and no images. In spite of that, questions including images and complex symbolic expressions are crucial in order to attain the coordination of semiotic systems. Moreover, items like multiple-choice or true-false ones cannot provide a complete information about students achievements. For example, devising a solution procedure for a problem, representing and describing it with words involve fundamental skills that should not be overlooked. Uncritical use of test items might also induce some high school teachers or students to neglect the skills related to mathematical proof. Thus users should be warned that prearranged-answer items cannot provide a complete evaluation of their achievements, and opportunities to deal with open-answer items should be provided anyway.

This could be achieved by means of resources allowing people to post files like Moodle's task or 'workshop'. Of course items of this sort cannot be evaluated automatically, but require more sophisticated patterns of evaluation or self-evaluation.

On the fall of 2006 at the University of Piemonte Orientale some 150 Biology, Chemistry and Environmental Sciences students have been offered more than three hundred quiz items covering all the topics of the ‘Introductory Mathematics’ course, from linear algebra to differential and integral calculus. On average each item has been dealt with by 34 individuals. More precisely, students split into two groups. About half of them visited the platform on a regular basis and tried to answer to a fair amount of items. The other half visited the platform occasionally and made just few attempts to answer to some item, and completed at most one set of them. The number of students regularly visiting the platform and attempting to answer to a reasonable amount of items has been far beyond our expectations. Their outcomes, although not significant from a statistical viewpoint, encourage us to go on with the experience and to expand and improve the offer for activities on the platform.

Interactions And Role-Play

The experience we are going to describe may be inserted in the framework of cooperative learning previously described. The experiment has been carried out in 2005-06 in the university of Eastern Piedmont in Italy. It has been organised by selecting two groups: an experimental group and a control one.

In our setting, the subject matter has been split into various sections. For each section rounds of different activities have been planned for the two selected groups.

The activities of the experimental group have been based on role-play. In each round each student has dealt with 3 topics:

- 1st: the student acts as a teacher, so he or she devises some questions as if he or she were to evaluate someone other’s learning outcomes;
- 2nd: the student answers to the questions proposed by a peer;
- 3rd: the student again acts as a teacher and checks the output (both questions and answers) of two peers.

At the end of each round, the tutors revise all the files produced and made them available to all the students.

The activities of the control group have been based on standard problem solving. Each member of the group was asked to autonomously solve problems provided by the staff (teacher+tutors) in a given time. Then the staff makes available solution patterns for self-evaluation.

An implicit selection of a third group has arisen: the passive users of the platform, who have at their disposal lecture notes, self-evaluation tests, other materials (worked-out problems, problems with hints for solution, FAQ), opportunity to contact the teacher, the tutors and other students.

The outcomes of the experiment have been collected at the end of the course by means of interviews, aimed at understanding how the activities carried out have affected the way of studying, which progress have been noticed by the students themselves, which role (among those played) has been considered particularly useful and why. The interviews have given evidence of many benefits due to peer-to-peer activities (see for example Albano, 2006, or Albano *et al.*, 2007): strengthening communication skills, critical enquiry and reflection; clarifying subject content through discussion; viewing situations from different perspectives; learning how to work as a team member; becoming actively involved in the learning process, learning to learn. In particular, looking at the benefits identified by the students for each role, we can summarize as follows.

The most appreciated role has been the first one, because it has allowed them to be in the teacher’s perspective, so getting able to understand the educational goals. Moreover, *to ask questions* have helped to study in a more critical and deeper way, with greater care, because it is not simple to pose a question due to the fact that there is no method to do that. At the same time, the request of a certain number of questions on a topic requires to range over all the programme, not only concentrating on the specific and restricted topic but also paying attention to all the other linked topics. It is also interesting to note that some students has used this role to make critical points clear (posing as questions exactly their own doubts). Finally we noticed some non-cognitive aspects such as the trend to pose non trivial questions, also for pride reasons, and this has required the mastery of the topics.

The second role, *answering questions*, has been considered useful because it has allowed students to appreciate topics usually neglected. It is commonly experienced by teachers the students’ quite general assumption about questions they consider *tricky* when posed at the exams. Some students have appreciated to receive from their colleagues some questions considered “tortuous” so that they have been forced to think about. Actually, if we see the papers produced by the students, there are no really tortuous questions, as well as there are not at the exams. Anyway the feeling of the students simply shows their familiarity with a flat and rote-learning style that is related to the lack of self-posed questions. In the same direction, we note that most of them have found questions that they did not think of before.

The role of the teacher who *checks the correctness* is not really much appreciated, essentially for two reasons: students do not feel themselves to be equal to this task or consider the task not useful because they think they surely will do well.

The role-play activities also affected students’ working methods. The students have acquired the habit of going into depth as a standard practice, and the habit of looking at something from more viewpoints (also through the comparison with other colleagues). This has changed attitudes toward studying, fostering the practice of reasoning rather than of learning by heart. The involvement in the activities proposed has given the students a sort of guidance for the organization of their study, providing time constrictions, topics to revise, indications of the relevant activities. Finally we want to note that some students have appreciated such kind of group activity also as training for their future work.

From a practical viewpoint, some management difficulties are to be mentioned. The experimental activities described require some work for revising students’ products and this has to be done *in itinere* as much as possible, in order to influence their further elaborations. So, on the basis of our experience, the availability of a staff, composed by a suitable number of tutors, is essential: maybe one tutor per 10-20 users could be appropriate. Of course the coordination among the teacher and the tutors has to be taken into account.

Communication And Semiosis

The activities described in the previous section are a good example of communication that involves the adoption of different registers (i.e. use-related linguistic varieties). The students have to understand each other, but also to convey some mathematical ideas. These two tasks may require different linguistic resources, and students have to switch between informal registers, in order to communicate each other as persons, and more formal ones, in order to describe mathematical ideas.

Looking at the files produced by the students through the activities, we can find a range of examples of conversion between different registers and semiotic systems. If we try to trace the evolution of the use of language by the students through the activities, we can say that at beginning the use of language is seemingly more formal, and in some sense more precise from the mathematical viewpoint. Actually, it is only a more rigid usage, due to the fact that students are not used to "talk of mathematics" and then their questions are standard (e.g. "What's the definition of a group?") so that the answers exactly conform to some piece of a book or lecture. Going on, students try to pose questions requiring some consideration for different topics or registers or semiotic systems, with the obvious consequence that answers cannot exactly conform to the style of a textbook. The presence of non standard questions has been increasing as much as the activities have gone on, with an average of 45% on the total amount of the questions. So for one thing, this is a good advance in mathematical thinking, for another thing , although they use a number of informal or even inaccurate expressions, students gradually improve their understanding of the meanings involved in mathematical expressions.

A platform, anyway, provides plenty of opportunities for designing communication situations involving the use of a wide range of linguistic resources.

More generally, ICT provides matchless opportunities for designing tasks involving conversion of semiotic systems, as defined in section 2.5. The following problem can be quite easily inserted as an item in different e-learning modules.

A problem like this (administered to Science freshman students) involves conversions between formulas, graphs and tables of values. It involves neither any advanced mathematical content nor any sophisticated use of semiotic systems, but it requires to coordinate some piece of mathematical knowledge and three different semiotic systems. Problems like this are hardly proposed in standard teaching activities, if they are based on paper-and-pencil or blackboard work only. Nevertheless, they provide unique learning opportunities from almost all the perspectives discussed in this chapter.

Affective Aspects: Students, Teachers And Mathematics

The use of an e-learning platform as a support to a standard lecture-based course also affects emotional aspects. Some investigations (Albano, 2005) have strongly pointed out students' expectations and beliefs on their relationship with mathematics and the teacher. The interviews have highlighted the importance of the role of the teacher as a tutor and as a guide for a proper use of technology. Otherwise, the computer may prove an obstacle if the work is not properly supported by the teacher, because of the risk of getting lost due to the "dispersiveness" proper of the technological tools. We underline that even from the first question the expectation of a wider contact with the teacher has been made explicit, and it remains unwavering through all the questionnaire. A considerable share of students actually expects an improvement in the relationship with the teacher, due to the increased opportunity to communicate provided by the technological tools. We suppose that this feeling of approaching (even if not physical) should be read as "*it is beautiful to know that there is someone*". In other words they greatly appreciate that the teacher is always at hand (by email for instance) if they wish or need. Through the platform the teacher is perceived closer, helpful, etc, and these factors have positive influence on the motivation to study, on the involvement in the course and on understanding. In almost 50% of the questionnaires the students refer to their expectation for wider, more frequent and easier opportunities to interact with the teacher. Such expectation is as strong as to be expressed anyway, independently from the question posed: we might be talking of either the course or their learning outcomes, or their relationship with mathematics, but in any case their expectation emerged in an almost "intrusive" way! At undergraduate level maybe this issue is felt as an important one because of the larger number of students per class compared to high school, which might weaken the relationship between teacher and student. So we can read their answers as a request for some contact with the teacher, who is felt remote and missing. Tools as those offered by the ICT not only make the students nearer to the teacher, but induce them to communicate in a less formal, less rigid, "warmer" way. In other words, the relationship between teacher and students becomes less asymmetrical.

Note that the improvement of the quality of the relationship between teacher and students greatly influence also the relationship between students and mathematics. Actually, the 44% of the students claim that the ICT-support, by itself, cannot change their feeling about mathematics, but most of them think that the teacher can strongly influence their relationship with mathematics anyway.

This is true of the quality of the course too: a teacher who doesn't love what he/she is teaching and who doesn't transmit passion to his/her students is the main, or maybe the only, factor that can "un-qualify" a course. On the other hand almost 20% of the learners states that a platform can improve the quality of a course since it allows to improve the relationship teacher-student because of a "direct contact" created (18,8%).

Anyway most of the learners (70%) expect to progress in mathematics learning and performance, thanks to the e-learning platform, because of the following main reasons:

- greater availability of contents/investigations/doubts/tests (37,2%);
- to be always in contact with the teacher (9,3%);
- course more interesting/practical/stimulant/new/involving (39,5%);
- easy, fast, deepen learning (23,3%).

Further investigations on such expectations have been carried out after attendance to the blended course in order to compare students' expectations and the actual outcomes (Albano, 2006, or Albano *et al.*, 2007). It has been found that the students' expectations have been met quite satisfactorily. The use of an e-learning platform really helps to create a relation with the teacher, that is quite lacking otherwise. We would like to underline that a teacher who uses a blended course has been considered as a teacher who takes care of the learning of his/her students, who wants them to be successful in their learning outcomes, who wants to communicate with them. Thus it positively affects students' motivation and then their outcomes: seeing the background activity of the teacher on the platform (such as materials updating, asynchronous interactions by emails and forum, etc) let students to feel encouraged and eager to learn. Moreover, being acquainted to communicate with the teacher can help to reduce the exam-related anxiety, which often cannot be overcome by the mastery of the subject only. Finally, the support offered by a blended course has proved an optimal help for students who failed previous exams. The benefits they got not only affected their cognitive and meta-cognitive state, but also improved their relation with mathematics.

FUTURE RESEARCH DIRECTIONS

We plan to go on with research on communication in an e-learning setting. of teaching for students with learning difficulties. This should take into account both the aspects more closely related to interpersonal communication and the specific features of the semiotic systems adopted in doing mathematics. This should involve the issues related to linguistic competence. Linguistic competence is most often than not ignored in research on mathematics learning at high school or undergraduate level.

We need a careful investigation of opportunities and limits provided by the use of multiple representation systems and by interpersonal communication. We also need to design activities supporting the acquisition of well-defined, advanced linguistic skills.

We want also to investigate how to create interactive, open-ended tasks engaging students in creative activities of construction, conversion and treatment of semiotic representations within different semiotic systems, in the setting of multiple-representation systems such as Computer Algebra Systems or Dynamical Geometry Systems. Actually, we already use multiple representations, but they are almost always pre-arranged by the teacher (e.g., test items involving graphs) and do not fully exploit the opportunity of asking the student to build the representations him/herself.

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