Solution of the Forward Problem of Electric Capacitance Tomography of Conductive Materials

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ABSTRACT

A feed-forward neural network is developed for solving the nonlinear forward problem of Electrical Capacitance Tomography (ECT). The ECT system is used in this work to determine the characteristic of the molten metal in lost foam casting (LFC) process. The metal-fill profile is one of the important factors that affect casting quality. The training data are generated by simulating different flow schematics of the molten metal during the casting process. Finite element method is used to generate the training data. Although a large amount of flow patterns and a considerable CPU time are required in the training phase, the trained network is characterized by simplicity and fast response. The nonlinear solution from the neural network can be combined with the Linear Back Projection algorithm to solve the inverse problem and generate the final images. The performance of the technique is compared with commonly used Linear Forward Projection (LFP) using sensitivity matrix, showing superiority in terms of both stability and quality of reconstructed images.

Keywords: Electrical Capacitance Tomography, Lost Foam Casting, Neural network.

1. INTRODUCTION

Electrical Capacitance Tomography (ECT) is used to image an object by measuring the mutual electrical capacitance between sets of electrodes mounted on its periphery. ECT system is superior in many cases over other tomography modalities due to its fast data acquisition speed, low construction cost, non-intrusive, non-destructive and safety. Reconstruction of the cross-section images in ECT system from the capacitance measurements is a nonlinear and ill-posed inverse problem. The image reconstruction process is ill-posed inverse problem because the number of unknowns (image pixels) is more than the number of known values (capacitance measurements). The nonlinear relationship between the physical distribution and the capacitance measurements makes the image reconstruction a challenging task [1]. There are two major computational problems in ECT: the forward problem and the inverse problem. The forward problem is to determine inter-electrode capacitance from the permittivity distribution by solving the partial differential equations governing the sensing domain. The inverse problem is to determine the permittivity distribution from the capacitance measurements. The result is usually presented as a visual image, and hence this process is called image reconstruction [2].

ECT system is used in many applications; Lost Foam Casting (LFC) process is one of these applications using tomography to describe the metal fill profile. It is very simple and cheap to cast very complex patterns by generating foam patterns as molds in LFC. In this process the molten metal decomposes the foam pattern and creates a casting in its shape [3]. LFC offers many advantages over conventional sand casting processes such as significant energy and environmental advantages, simplified production techniques and reduced environmental waste due to binder system emissions and sand disposal, beside the low cost. The process is well known for casting complex geometries, small details, and smooth surface finishing requirements.

A better understanding of the characteristics of the molten metal inside the foam pattern is needed to reduce the fill related defects and to improve the final casting [4]. Much effort has been put into developing a wide variety of imaging techniques for industrial process applications over the past two decades. For example, X-ray techniques [5] are presently used to assess the filling characteristics of the liquid metal. However, X-ray methods suffer from the natural hazards of radiation and size of the equipment and it is very expensive compared to the ECT system.

Most of image reconstruction algorithms are based on iterative techniques due to the nonlinear relation between the sensing field and the physical distribution [6-8]. In iterative techniques, the error between measured and calculated capacitance is minimized by updating the reconstructed image accordingly. A linear model, called sensitivity matrix, based on linearzing the relation between the physical property and the measured capacitance is generated to update the image.

The sensitivity matrix is generated by dividing the domain of interest into small pixels then the capacitance data are obtained as a linear sum of different perturbations composing the overall distribution. Although the use of the sensitivity matrix increases the speed of solving the forward and the inverse problems, the results are blurred and poor due to the linear approximation. Another big problem associated with the use of conductive material in the LFC application is the shielding. Having the metal in front of any sensor makes it blind to any change behind the first level.

To overcome such limitations, a nonlinear forward solver based on intelligent Feed-Forward Neural Network (FFNN) has been developed in this paper. The use of FFNN combines the advantages of solving the forward problem nonlinearly with high speed and high accuracy. The training data are generated based on different models developed to describe the behavior of the molten metal during the casting process [9-11]. The characteristic of the molten metal mainly depends on the temperature of the molten metal, geometry of the casting, type of the foam beads, and the location of feeding of the molten metal inside the foam pattern. There are three models describing the metal file profile. All of them are simulated by finite element method to calculate the capacitance measurements related to all different distributions. To increase the accuracy of the neural network some random metal distributions are generated and used to train the neural network.

The organization of this paper is as follows. Section II is an overview of the ECT technique. The structure of the capacitive sensor is stated, followed by the calculation of the sensitivity matrix. The new feed-forward neural network for solving the forward problem is described in Section III. The analysis of the results is discussed in Section IV, and Section V contains the final conclusions.

2. ELECRICAL CAPACITIVE TOMOGRAPHY

ECT Hardware

Generally, in the ECT system the capacitive sensors are arranged as an array of *n* electrodes mounted around the periphery of the imaging area. The electrodes are externally shielded to eliminate the stray capacitance effects [12]. All independent mutual capacitance measurements are measured between transmitter electrode connected to the source signal and the other receiver electrodes connected to the ground. Subsequently, the next electrode is made as a source and the same measurement process is employed [13], that means if the ECT system consists of *n* electrodes the number of the independent measurements is $n(n-1)/2$.

The sensor array used in this study consists of 12 electrodes mounted uniformly around the foam pattern (imaging area) as shown in Figure 1. The foam pattern is embedded inside compressed sand in a grounded flask. The flask is connected to the earth to prevent disturbance from outside environment.

ECT Model

The forward problem solution determines the capacitance measurements of the ECT sensor given the grounded metal distribution in the region of interest. The relationship between the capacitance measurements and the permittivity distribution can be characterized by Poisson's equation [4]:

$$\nabla.\left(\varepsilon\left(x,y\right)\nabla\varphi\left(x,y\right)\right)=-\rho\left(x,y\right) \tag{1}$$

where $\varepsilon(x,y)$ is the permittivity distribution in the sensing field, $\varphi(x,y)$ is the electrical potential distribution, and $\rho(x,y)$ is the charge distribution.

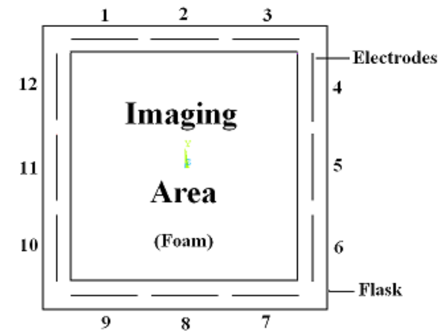


Figure 1: Cross-sectional view of ECT electrodes system with 8 sensors

The free charges in the imaging domain are zero, which means $\rho(x,y)=0$ and Poisson's equation is converted to Laplace's equation applied on the boundary conditions.

$$\psi^{i}=\begin{cases} V_c & (x,y)\subseteq all\ \Gamma_i \\ 0 & (x,y)\subseteq all\ \Gamma_k (k\neq i)\ and\ (x,y)\subseteq(\Gamma_s+\Gamma_{pg}+\Gamma_e) \end{cases} \tag{2}$$

where $\Gamma_1,\Gamma_2,....,\Gamma_n$ represent the spatial locations of the *n* electrodes, Γ_s is the sensor screen and Γ_e represents the spatial locations of the *m* elements where the grounded metal will replace the foam. The electrical potential distribution can be calculated numerically by using the finite elements method (FEM). The mutual capacitance can be calculated by using Gauss's law after calculating the charges on the received electrode:

$$C_{ij}=\frac{Q}{V}=\frac{1}{\Delta V_{ij}}\oint_{\Gamma_j}\varepsilon(x,y)\Delta\phi(x,y)d\Gamma_j \tag{3}$$

where ΔV_{ij} is the potential difference between the source and the detector electrodes, Q is the electrical charges on the electrode, and Γ is the electrode surface.

In all the iterative and the non-iterative reconstruction techniques, the nonlinear relationship between the capacitance measurements and the metal distribution is approximated by a linear function [2-4]. The discrete form of this linear relationship can be expressed as

$$\Delta C_{mx1}=S_{mxn}\Delta H_{nx1} \tag{4}$$

where S is a sensitivity matrix, ΔC is the capacitance measurements vector, ΔH is a vector representing the change in metal distribution and the boundary conditions due to change in the metal fill, *m* is the number of the measurements, and *n* is the number of elements inside the imaging area which equals to 256 in the model used in this paper. Figure 2 shows the finite element model of the metal fill problem by using ANSYS software. It shows the domain of solution divided into small elements. In this model the foam pattern, the imaging area, is divided into 16 × 16 grid generating 256 pixels, and the total number of finite elements in the whole model is 1832.

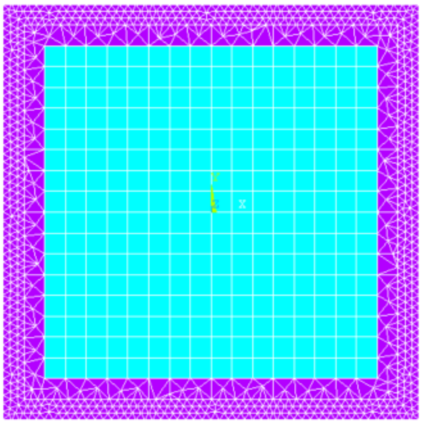


Figure 2: Finite element model.

Forward Problem Solution

The forward solver aims to compute the response of all the electrodes by changing one element k from foam to grounded metal and keeping the rest of the elements as foam. The importance of fast forward solutions is manifested when iterative algorithms for image reconstruction are used. In iterative algorithms, the image obtained from reconstruction is updated by minimizing the error between the measured capacitance data and the forward solution for a predicted permittivity distribution. This process is repeated iteratively until a predefined criteria is met, so multiple forward solutions become necessary. Obtaining explicit forward solutions from (1)–(3) via brute-force numerical techniques is a time-consuming task, and hence alternative techniques must be explored.

The output of the forward solver is the sensitivity matrix. The sensitivity model is an implementation of the superposition theorem, in which the forward solution is obtained as a linear sum of capacitance measurements from perturbations in permittivity distribution. A finite element model generated by using ANSYS is used to compute the mutual capacitance measurements $C_j(k)$ $k=1,2,\ldots$ for all the sensors. Finally, typical sensitivity matrix distributions are respectively calculated according to (5)

$$S_i(k)=\frac{C_i^F-C_i^k}{C_i^F-C_i^M} \qquad (k=1,2,\ldots,n;\, i=1,2,\ldots.) \tag{5}$$

where n is the number of elements inside the foam pattern, C_i^F is the capacitance measurement when the imaging area is completely foam, C_i^M is the capacitance when the metal fills the foam pattern completely, and C_i^k is the capacitance value after replacing element k in the foam pattern by grounded metal. The sensitivity matrices between electrode one tol four are shown in Figure 3. The response is very high around the sensor and decreases gradually by moving further from the sensor set. The rest of the elements around the imaging area have zero sensitivity because they represent the sand elements where there is no metal.

The normalized measurements vector is obtained by using the following equation

$$C_N=\frac{C_i^F-C_i}{C_i^F-C_i^M} \tag{6}$$

where, C_i^F , C_i^M are the measurement vectors when the imaging area is entirely filled by the foam and the metal, respectively. C_i are the measurements corresponding to a certain grounded metal distribution. For any sensor, the high normalized measurement means that the grounded metal is very near to that sensor.

The most common method used to solve the forward problem in image reconstruction process is linear forward projection (LFP) [12]. The LFP method is based on the sensitivity model. Based on this model, the mutual capacitance as a function of permittivity distribution can be written as (4). This technique suffers in a smoothing effect and lack of accuracy due to the linearization of an inherently nonlinear problem of electrical tomography.

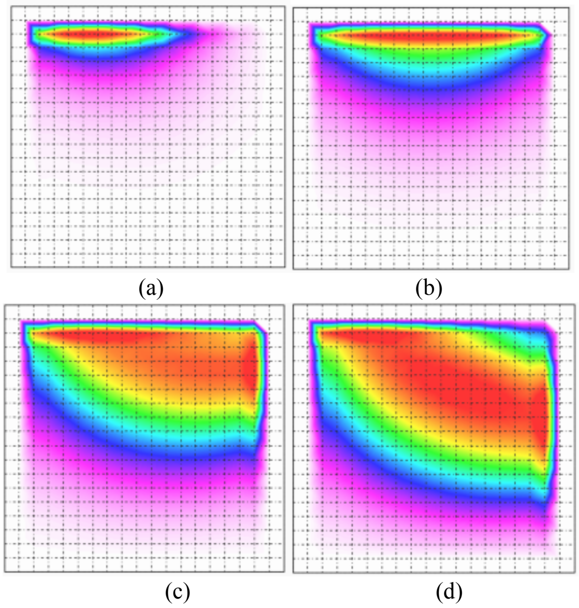


Figure 3: Four sensitivity matrices for a symmetrical capacitive sensor array system, all dimensions in X and Y directions are in inches. (a)Sensitivity matrix between electrodes 1 and 2.(b) Between electrodes 1 and 3 (c) between electrodes 1 and 4. (d) Between electrodes 1 and 5.

3. Feed-Forward Neural Network (FFNN) Solver for the Forward Problem

Artificial NNs are composed of simple processing elements called neurons arranged in different layers and connected with each other by weighted links [13]. The weight of each link represents the strength of the connection between two neurons. NNs play an important role in various applications and possess the property of being a universal approximator, i.e., for any function of arbitrary degree, there is a feed-forward NN able to approximate it. NNs are considered an attractive choice for modeling nonlinear and complex problems because of their robustness, ability to withstand noise, their universal approximation property, and ability to predict and extrapolate information hidden in the training data, in a process known as NN learning [14]. An important aspect in the image reconstruction process is the speed in the prediction once trained without need for linearization assumptions.

A multilayer Feed-Forward Neural Network (FFNN) consists of a number of neurons organized in multiple layers is shown in Figure 4. Each neuron is connected with a weight to all neurons in the adjacent layers. The value of each weight represents the relevance of the particular connection in the network structure. Each neuron output is mapped to a transfer function. A sigmoid function is usually employed to map unbounded data to the bounded range of the transfer function. Different sigmoid functions can be used with different ranges.

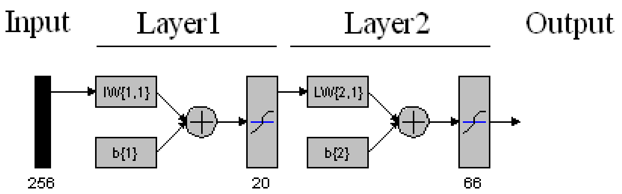


Figure 4: FFNN with multiple layers in matrix form.

Using FFAA is introduced in this work to solve the forward problem. The proposed neural network uses the metal distributions itself as inputs and the normalized capacitances as outputs. It consists of tow layers with 20 neurons in the hidden layer. Thus for ECT system with 16x16 image grid and 12 electrode plates, the size of the neural network would be 256 inputs neurons and 66 outputs.

Training Data

The training data are generated based on simulating the characteristic of the molten metal inside the foam patterns during the casting process. This characteristic is related to the modes of foam decomposition where lost foam patterns can decompose by different physical mechanisms. These modes depend on local conditions that develop as the mold fills as well as some other parameters such as type of the foam beads and gating source where the metal is injected [9]. There are three modes for the foam decomposition [10, 11]: 1) Contact mode, in which the liquid metal presses directly against the foam as shown in Figure 5. 2) Gap mode, caused when polymer vapor bubbling through the liquid metal accumulates along an upper segment of the flow front, opening a finite gap between the liquid metal and the decomposing foam as shown in Figure 6. 3) Collapsed mode, Figure 7, which occurs in regions of the pattern that contain large amount of connected, inter-bead porosity.

The training data are generated using finite element software package (ANSYS) to simulate the foam decomposition modes and the molten metal flow in every mode. The metal starts at the gate and grows up based on the mode of decomposition. Each frame represents the distribution of the metal at this time. Based on this distribution the capacitance measurements are computed and normalized according to equation (6). Different gating locations are considered during creating the data to simulate all possible distributions of the metal during the casting process. To increase the accuracy of the neural network many random distribution are generated to increase the number of data used in the training phase. 3000 metal distributions are used in training the feed-forward neural network implemented in this work. Figure 8 shows the metal distributions simulating the contact mode while the metal is fed from a gate on the top.

For example, the normalized capacitance data from the finite element simulation for the second and last distributions in Figure 8 are shown in Figure 9. In the first distribution, a small piece of metal around electrode 2 causes the normalized capacitance between electrode number 2 and the other electrodes to be the maximum value almost one. The other measurements are very small because the metal is not in their sensing area. While, the measurements in Figure 9(b) coming by having half of the imaging area filled by metal are very high, equal one, from electrodes 1-3, 4 and 12.

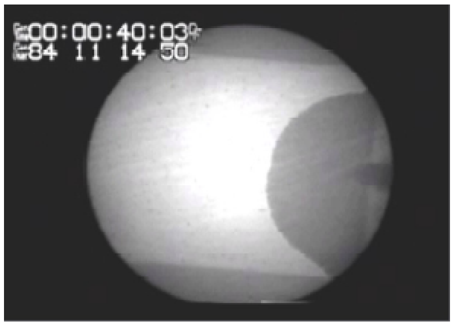


Figure 5: X-ray image for filling 8-mm thick pattern. The regular, smooth flow front is characteristic of contact mode.

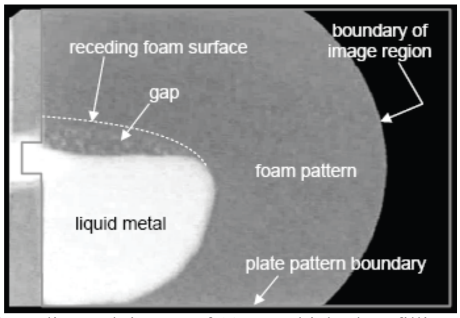


Figure 6: Radiogragh image of 12-mm thick plate filling from the side showing the gap mode.

Figure 7: X-ray image for filling 8-mm thick pattern. The rapid, irregular metal flow is evidence of collapse mode.

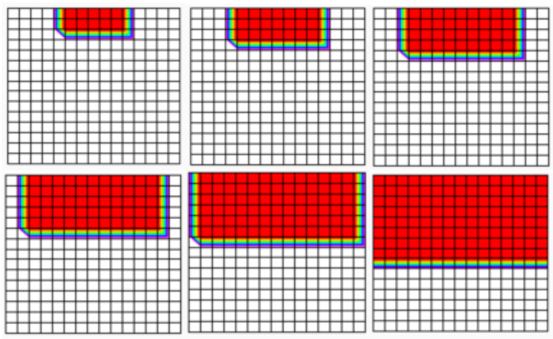
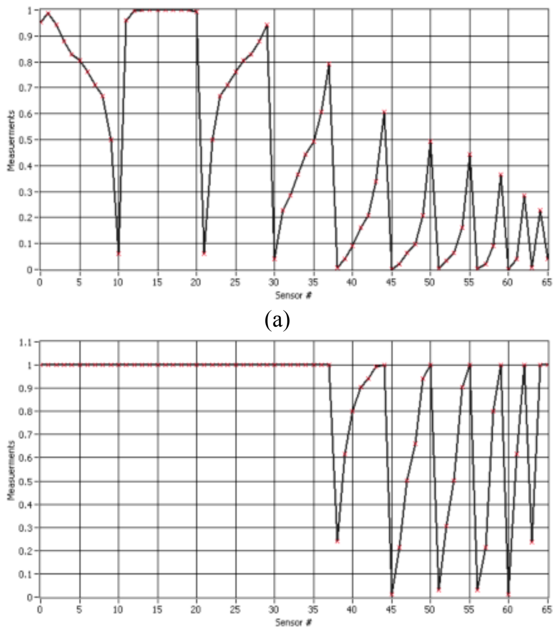


Figure 8: Metal distributions simulating the contact mode.



(a)

(b)

Figure 9: Normalized capacitance measurements for metal distributions in Figure 8, (a) First distribution (b) Last distribution.

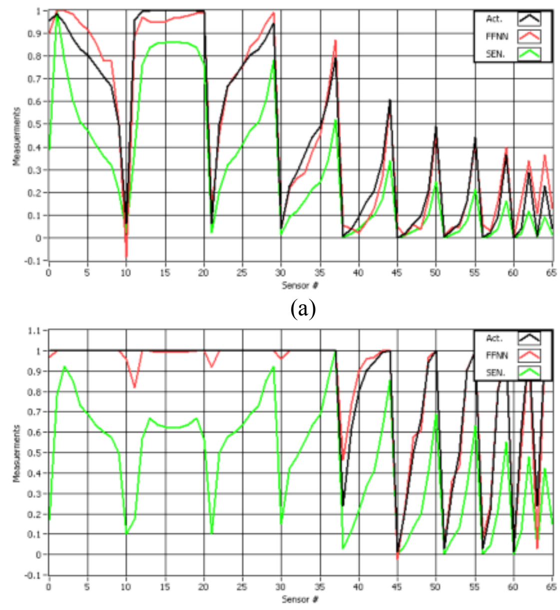
4. Experiment and Results

An ECT system consisting of 12 electrodes is used to test the proposed technique to solve the nonlinear forward problem. The arrangement of the 12 electrodes is described in Figure 1. The square flask is 20x20 inch while the imaging area is 16x16 inch and the size of the electrodes is 4 inch. The imaging area is divided into 256 elements. A Feed-Forward Neural Network has been trained to solve the forward problem. The inputs of the neural network are the metal distributions and the normalized capacitances from all the sensors are the outputs. Thus for ECT system with 16x16 image grid and 12 electrode plates, the size of the neural network would be 256 inputs neurons and 66 outputs. The network was trained with the generated data explained in the pervious section. It took about 2 hours and 2500 iterations to train the network with mean square error (MSE) performance of 0.000277 (RMS error 1.94%) on a Windows XP machine with Pentium D processor (3.8 GHz) and 1 GB memory.

Both the FFNN and the LFP predictions for the second and the last distributions in Figure 8 are plotted in Figure 10(a) and 10(b), respectively, and compared to the actual measured capacitance of the same metal distribution used in prediction. The normalized capacitance data generated using NN is more correlated to the measured capacitance than LFP. The mean square error for the data generated using the second distribution from the NN is 0.00367 while using the LFP algorithm is 0.047. The result from the NN is much better than the LFP in Figure 10(b) because of the effect of the shielding. The neural network takes care of the nonlinearity in the response of the sensors especially when there is a big piece of metal shielding completely the sensor compared to the LFP which assumes linear response. The MSE in the second case related to the second distribution is 0.0029 for the neural network and 0.1961 for the LFP algorithm.

5. CONCLUSION

A new technique based on multilayer feed-forward NNs for solving the nonlinear forward problem in Electrical Capacitance Tomography has been introduced in this work. Commonly used LFP forward problem solution is used to compare the results from the proposed neural network system. Results showed superiority of the proposed technique in terms of accuracy and overcoming the problem of excessive time and computer resources necessary when using brute-force numerical techniques for the forward problem. Using NN takes care of the nonlinearity of the system and eliminates the shielding effect of the grounded metal. The described technique is fast and easy to implement in any iterative reconstruction algorithm. The main limitations of the technique are the training time and prior information required. Sufficient training data has to be collected.



(a)

(b)

Figure 10: Predicted capacitance vector using both NNs and LFPs compared to measured capacitance of the same permittivity distribution.

6. ACKNOWLEGMENT

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