A New Method of Frequency Tracking Based on Modified Adaptive Notch Filter

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Abstract –In this paper a new method of frequency tracking based on modified adaptive notch filter is proposed to solve the problem of dynamic frequency tracking of low capacity power system, the ANF equations is discretizated, the influence of the ANF parameters on frequency tracking is analyzed, the band-pass pre-filter and the low-pass post-filter are applied. With applying iterative updates synchronous with sampling cycle, the modified ANF tracks the frequency of a power system and approaches the signal frequency quickly and precisely. The simulation results demonstrate that the method is characterized of good frequency tracking, static and transient performances and immunities to harmonics, noises, and dc current component as well.

Key words: Adaptive notch filter, frequency tracking, power system, iterative update

Ⅰ. INTRODUCTION

A power system runs at its nominal frequency generally. The imbalance between the power system and its loads will result in a frequency deviation from the nominal value. It is desirable to detect or estimate the network frequency quickly, precisely and reliably in many engineering applications such as parameter measurement, electric energy quality control, failure diagnosis, reactive power and harmonic detection and compensation.

The low capacity power systems, which are widely used in ocean freight vessels, special construction vessels and offshore drilling units, are characterized by their capacities around 10 - 50MW. When the large power electronic devices are started, the low capacity power systems will be seriously affected which not only caused the parameters in the network changed frequently, but also produced a lot of harmonics and noises. Therefore, it is more difficult to conduct dynamic detection and estimation of frequency quickly and accurately for a low capacity power system than for a high capacity power system.

There are a lot of methods to detect and estimate frequency [1, 2, 3, 4, 5], such as zero-crossing, discrete Fourier transform (DFT), Kalman Filter, phase-locked loop (PLL), least square fitting (LSF), wavelet transform (WT) and Helbert-Huang transform (HHT) et.al. The zero-crossing method detects frequency by measuring the time interval of the two consecutive zero-crossing points of the signal wave, the advantage of the method is easy to be realized, while the disadvantage of the method is its poor measuring accuracy, and sensitive to the influence of harmonics, noises, dc components, etc. The DFT algorithm estimates frequency using sampling sequence, but it is very sensitive to the distortion of the input signal. Kalman Filter algorithm estimates the variables of frequency status optimally with basic Kalman Filter equations, but it can not ensure its robustness to the change of equation parameters. Although PLL offers relatively good performance,

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This project is sponsored by Shanghai Education Committee. Item number is 06FZ020

the requirement of using a voltage-controlled oscillator will cause a complex structure. The HHT algorithm is mainly applied for complicated and unsmooth signals. Since it is an adaptive analysis method, the transcendent knowledge is not necessary. In the HHT algorithm, the decomposition is largely dependent on the measured signal and has high time-frequency resolution, but the boundary problem and principle of decomposition selection problem must be investigated further more.

An ANF can change its notch frequency according to the input frequency. It was first proposed by Regalia [6] used in frequency tracking, which is a lattice-based discrete time model. Later Bodson [7] used it in the continuous time case and developed a continuous function model. The algorithm was modified by Hsu successfully [8], resulting in a big improvement in its performance. Mojiri [9] made some modifications to the existing equations and developed the new enhanced equations, a lot of simulations on parametric trajectory were conducted, and the algorithm was gradually developed. On the base of lattice-based notch filter, a new approach was proposed by Zhang Shiping. The approach consists of two-stage adaptive notch filters. The first stage eliminates harmonics and enhances the fundamental waveform, and the subsequent notch filter is fed with down sampled version of enhanced signal obtained from first stage.

In this paper, the adaptive filtering equations are discretized based on Hsu's adaptive filter, a band-pass pre-filter and a low-pass post-filter are applied to filter out the disturbances of harmonics and noises in input signal. With the iterative updates synchronous with sampling cycle, the modified ANF tracks and estimates the frequency, and approaches the signal frequency rapidly and accurately.

Ⅱ.The PRINCIPLE AND the FLAWS OF the ANF

An ideal notch filter is an infinite impulse response (IIR) filter, and obeys the frequency response which offers the gain of 0 at its notch frequency and 1 at any other frequency. A narrow and deep notch is formed at the notch frequency, which removes the notch frequency and retrieves any other frequency else. The transfer function of an ideal notch filter can be expressed as follows:

$$H(z)=\sum_{i=0}^{M}a_i z^{-1} \Big/ \left(1+\sum_{i=1}^{N}b_i z^{-1}\right)$$

Considering a special cases: suppose z_1 zero locates on the unity circle in the first quadrant and p_1 pole locates on the circle's radius nearby the zero, while z_2 and p_2 are zero and pole in the fourth quadrant respectively, which are symmetric about z_1 and p_1, the notch filter becomes a single notch frequency notch filter, and its transfer function can be expressed as:

$$H(z)=(z-z_1)(z-z_2)/(z-p_1)(z-p_2)$$

Where, $p_1 = (1-u)\, z_1$, and $p_2 = (1-u)\, z_2$. The smaller the u, the closer the poles locate to the zeros, and the sharper and narrower the notch of the frequency response is. The bode diagram of the transfer function is shown in Fig. 1.

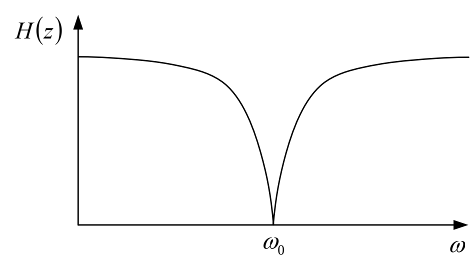

Fig. 1 Bode diagram of a typical notch filter

It is obviously that this kind of notch filter is able to measure sine periodical signal of network, which has a fixed angle frequency ω_0. But its measuring performance is seriously deteriorated by the variation of the network frequency, hence the typical notch filter is not suited for detecting the dynamic network frequency.

Ⅲ. THE DISCRETE ANF MODEL

Suppose that $y(t)$ is the input ac signal to be measured (the ac voltage or the ac current)

$$y(t)=A_0+\sum_{k=1}^{\infty}A_k\sin(k\omega_0 t+\varphi_k)+n(t) \tag{1}$$

Where, A_0 is dc component, A_k is the amplitude of kth harmonic, ω_k is the phase offset of the kth harmonic, $n(t)$ is noise. Hsu's differential equations [8] of dynamic response in frequency tracking are given by

$$\left.\begin{aligned}&\ddot{x}(t)+2\xi\omega(t)\dot{x}(t)+\omega^2(t)x(t)=\omega^2(t)y(t)\\&\dot{\omega}(t)=-\gamma x(t)\left(\omega^2(t)y(t)-2\xi\omega(t)\dot{x}(t)\right)\end{aligned}\right\}\quad(2)$$

Equation (2) describes a non-linear continuous variable model, where $x(t)$ is the indirect state variable, $\omega(t)$ is the angle frequency to be tracked, $\omega = 2\pi f$, f is the fundamental frequency of the network to be tracked and measured, $y(t)$ is the instantaneous signal measured (voltage or current), ξ is the damping ratio, γ is the adaptation gain.

The dynamic adaptive frequency tracking is mainly characterized by (2), and let the error of network frequency ripple to be:

$$e(t)=\omega^2(t)y(t)-2\xi\omega(t)\dot{x}(t)\quad(3)$$

Thus, equation (2) can be expressed by:

$$\left.\begin{aligned}&\ddot{x}(t)+\omega^2(t)x(t)=e(t)\\&\dot{\omega}(t)=-\gamma x(t)e(t)\end{aligned}\right\}\quad(4)$$

Equation (4) demonstrates that the dynamic tracking of network frequency can be achieved by calculating the error of network frequency ripple continuously and updating the calculated frequency value.

A. Discreterization

In order to construct the numerical ANF, suppose that the sampling step is a unity time and the indirect variables *y1~y9* are set. The equations (2)-(4) are discreterized with Euler's formula, forming the differential equations summarized as follows:

$$\left.\begin{aligned}&y_1(n)=\omega(n-1)\omega(n-1)\\&y_2(n)=y(n)y_1(n-1)\\&y_3(n)=\omega(n-1)y_7(n-1)\\&y_4(n)=y_2(n)-2\xi y_3(n)=e(n)\\&y_5(n)=y_1(n)y_8(n-1)\\&y_6(n)=y_4(n)-y_5(n)\\&y_9(n)=-\gamma y_4(n)y_8(n-1)\\&y_7(n)=y_7(n-1)+y_6(n)\\&y_8(n)=y_8(n-1)+y_7(n)\\&\omega(n)=\omega(n-1)+y_9(n)\end{aligned}\right\}\quad(5)$$

Equation (5) implements the iterative updating of dynamic frequency tracking for the *n*th sampling $y(n)$ of the input signal. Note that in the current updating (the *n*th), the current variable value (the *n*th) can be replaced by the last result of calculation (the (n-1)th), and the calculating sequence in (5) can not be altered. In (5), $y_6(n)$ is $\ddot{x}$, $y_7(n)$ is $\dot{x}$, $y_8(n)$ is x, and . is the angle frequency of the fundamental frequency f in the network to be measured, $\omega = 2\pi f$. The all operations contained in (5) are addition and multiplication, and can be conducted easily and quickly, meeting the requirement of real-time online calculation.

The block diagram of the ANF frequency tracking algorithm can be obtained from (5), which is shown in Fig. 2

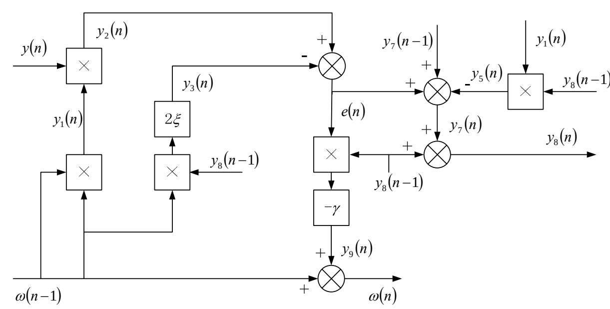

Fig. 2 Block diagram of the ANF frequency tracking algorithm

From Fig. 2 we can see that the dynamic error $e(n)$ at the *n*th sampling can be calculated by the current input $y(n)$ and last frequency tracking result $\omega(n-1)$, and thus the current frequency $\omega(n)$ can be estimated. In fact, the adaptive frequency tracking means that the current frequency value

$\omega(n)$ is estimated by the current iterative updating with the last frequency tracking result $\omega(n-1)$. The fundamental frequency will be tracked precisely after 2 or 3 fundamental cycles with the iterative updating of the variables.

B. Discussion About ξ and γ

There are two parameters ξ and γ in the equation (2). ξ is damping ratio. The increase of ξ will decrease the overshoot of the frequency tracking curve and the appearance of the periodic oscillation, but will reduce its response speed. γ is adaptation gain. The increase of γ will reduce the transient time of frequency tracking and increase the response speed, but will increase the instability of the system.

Increasing ξ and γ randomly will not be able to improve the performance of the frequency tracking. For example, the response speed can be enhanced by increasing γ, while the overshoot and the oscillation of the frequency tracking curve will be increased. And in this situation, ξ should be increased to reduce the overshoot and alleviate the oscillation, which will decrease the response speed in reverse. That is why effective frequency tracking is implemented on by the reasonable selection of ξ and γ. For the fundamental sinusoidal signal, the asymptotical stability is give by:

$$\frac{A_1^2\gamma}{2} < 1 \qquad (6)$$

It is not easy to obtain the optimal tracking wave. Because different harmonics affect the frequency tracking of (2), the optimal result will obtained only with the reasonable modification of parameters of ξ and γ. We get the following experience results from experiments and analysis:

- When the total harmonic distortion (THD) is below 5%, the value of ξ should be in the range of 0.4 - 0.5 and the value of γ should meet 0.55< $A_1^2\gamma$ <0.65, which will achieve the best result of frequency tracking.
- When the value of THD is in the range from 5% to 15%, ξ should be valued from 0.5 to 0.62, and the value of γ should meet 0.65< $A_1^2\gamma$ <0.7.
- When the value of THD is in the range of 15% to 40%, ξ should be valued from 0.62 to 0.73, and the value of γ should meet 0.7< $A_1^2\gamma$ <0.74.
- When the value of THD is above 25%, ξ should be valued about 0.75, $A_1^2\gamma$ about 0.76.

Ⅳ. PRE-FILTER AND POST-FILTER

When the ANF is used for frequency tracking, if THD is above 5%, an error of frequency will be produced along with a small oscillation, which will become serious as the contents of harmonics increase. To solve the problem, a pre-filter and a post-filter are fixed at the input port and the output port respectively, to eliminate or attenuate the error and oscillation. The pre-filter, a band-pass, is used to eliminate harmonics and noise and enhance the fundamental component. And the post-filter, a low-pass filter, is used to eliminate the ripple of frequency tracking. The two filters presented in the paper are finite impulse response filters (FIR), their structure are shown in Fig. 3.

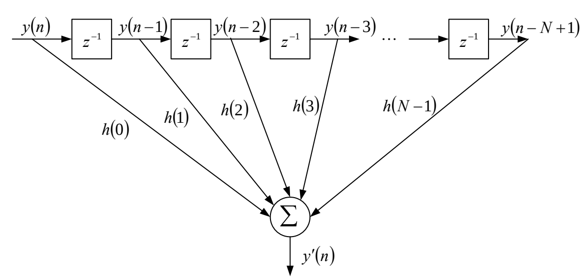

Fig. 3 Structure of the pre-filter and post-filter

The FIR is featured by easy to be designed, good stability, simple realization and linear phase as well. Its output is dependent only on its current and past input, free of its pass output. For the band-pass pre-filter, the cutoff frequencies are set to 20Hz and 100Hz, the number of filter coefficients *N* is given by 150. For the low-pass post-filter, which is used to output frequency tracking, the cutoff frequency is set to 150Hz, and number of filter coefficient *N* 30. The values of coefficients are determined by simulation.

The test wave forms of the input and output of the pre-filter are shown in Fig. 4, where the fundamental frequency is 50Hz, and 3rd and 5th harmonic component accounts for 35% and 20% respectively. It is obvious that the high order harmonics are filtered out by the pre-filter, while the fundamental wave remains its original waveform. However, some delay is inevitably produced by the numeric filter, the delay in Fig. 4 is about 60 sampling points (about 4.7 ms). In order to shorten the delay time, the length of filtering network (i.e. the number of the filtering coefficients *N*) should be reduced properly.

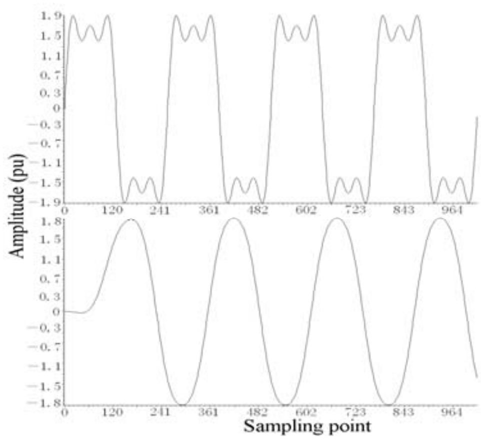

Fig. 4 Test wave forms of the input and output of the pre-filter

Ⅴ. SIMULATION

To verify the performance of the modified ANF of dynamic frequency tracking, a mixed input signal is generated by superposing 5% dc component, and some harmonics of 33% 3rd, 20% 5th, 13% 7th and 11% 9th to the fundament. Calculation shows that THD reaches 41.3%. Hence, ξ is set 0.75 and ($A_1^2\gamma$) 0.76. The frequency tracking curve resulted in the simulation test is shown in Fig. 5, with the tracking time is about 0.053 second (2.65 cycles).

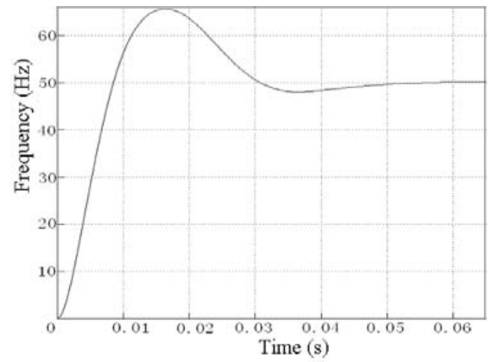

Fig. 5 Simulation test curve of network frequency tracking

A. Frequency Tracking under the Step Input

The initial fundamental frequency of the input is tuned to 50Hz, and it is stepped up to 60Hz at 0.2 second, and stepped down to 40Hz from 60Hz at 0.4second. It can be seen from the curve ② in Fig. 6 that the response time is 2 - 3 cycles, the overshoot is under 1% with 2 - 3 oscillations.

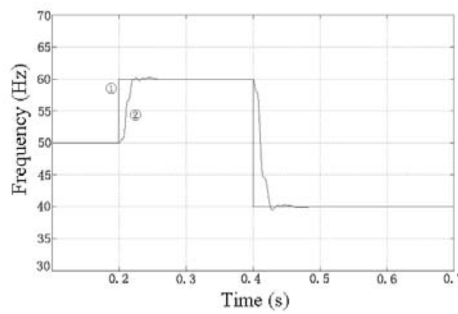

Fig. 6 Frequency tracking response to step change input

B. Frequency Tracking under Ramp Input

The initial fundamental frequency of the input is tuned to 40Hz, and it is increased by 50Hz/s at 0.2 second. The curve ② in Fig. 7 exhibits the frequency tracking response to the ramp input. It can be seen from Fig. 7 that the ramp input ① is closely followed by the frequency tracking ②.

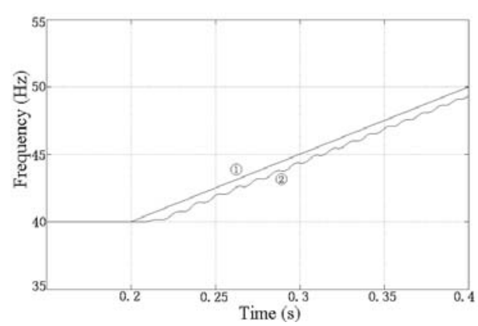

Fig. 7 Frequency tracking response to ramp input

C. Frequency Tracking under Sharp Phase Change

The initial fundamental frequency of the input is tuned to

50Hz, and a sharp phase change of 60 degree is given to the input at 0.17 second. It can be seen from Fig. 8 that the frequency tracking begin to deviate from 50Hz at 0.17second, and deviation reaches its maximum (about 1Hz) after 0.02second and then decreases and vanishes. The entire transient procedure lasts about 0.05second (2.5 cycles). The largest deviation is controlled below 2%, and quick frequency tracking is achieved with good dynamic performance.

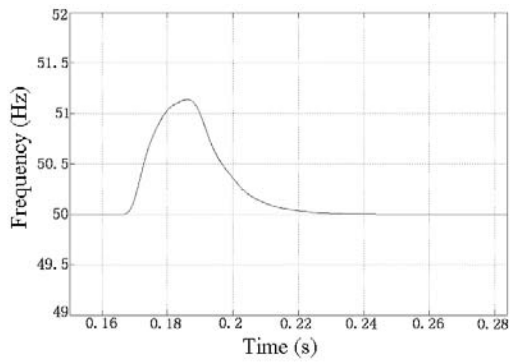

Fig. 8 Frequency tracking under sharp phase change

D. Frequency Tracking Under Periodical Frequency Variation

The fundamental frequency is tuned to ($45 + 5\sin 60\pi t$) Hz, varying periodically with time going. From Fig. 9 it can be seen that the frequency tracking ② fluctuates with the real frequency value ① up and down, with a bit of delay and an error less than 5%.

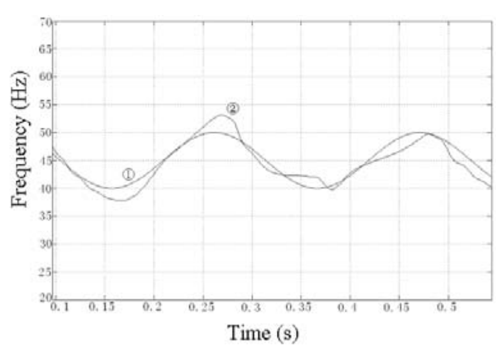

Fig. 9 Frequency tracking under periodical frequency variation

Ⅵ. CONCLUSION

In this paper, the principle of ANF and the method of setting parameters of notch filter is analyzed concretely. The modified ANF was proposed to implement the dynamic frequency tracking and estimating of the network. The FIR filters were utilized as the pre-filter and post-filter, which achieved the immunity of frequency estimating to various disturbances such as harmonic, noise and DC component, obtaining fine dynamic or steady performance. Compared to other methods of frequency estimating, this method is characterized by simpler structure, more accurate and faster calculation and strong immunity. The method was verified effectively by practical experiments.

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